Coupling Mechanisms for Multi-Fingered Robotic Hands with Skew Axes

Taher Deemyad, Neda Hassanzadeh and Alba Perez-Gracia

Abstract The design of wristed, multi-fingered robotic hands for specific tasks usually leads to solutions in which the position of the joint axes have little to do with those of anthropomorphic hands. Joint axes tend to be skewed, which makes difficult the location of the physical joint along the axis. In addition, transmissions that work smoothly with parallel or perpendicular axes may not work well with skew axes. In this work, we present the optimal design of a simple five-fingered hand, consisting of a wrist joint and five fingers with a revolute joint each. This hand is designed for a two-position task and optimized first for dimensions and to avoid self-intersection. Then a transmission to couple the degrees of freedom is also optimized, in order to create an underactuated, pick-and-place hand able to create 5 contact points while being driven with single actuation.

1 Introduction

A wristed, multi-fingered hand consists of a single serial chain spanning several serial chains, with a kinematic tree topology. Kinematic analysis of tree topologies for applications in modular robots and robotic hands can be found in [12], [14], and [2].

The design of multi-fingered hands has been studied from the point of view of selecting the right number of fingers and the mobility and connectiviy of each finger for grasping and manipulation in [9], [15], [4] and [6]. In [13], algorithms were created to combine properties of mobility and force closure for the type synthesis of general hands.

The creation of the robotic hand once the hand topology has been selected follows two more steps. First, the joints need to be placed so that the hand can reach

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the desired task, which is a dimensional synthesis problem. Kinematic dimensional synthesis of tree topologies presents particular challenges that have been explored in [11], [10] and [5]. Dealing with a coordinated action of all the fingertips leads to a *simultaneous task* for all end-effectors, in which case relative motion among them has to be considered too.

Obtaining the type and number of joints and the location of the joint axes has to be followed by a process in which the designer needs to decide where to physically implement the joints along the axes. This process does not change the kinematic task, but has a great influence in the hand dynamics, force transmission, self-intersection and overall hand size and inertia. Some of these parameters can be designed with an optimization process such as the one in [18] and automatically implemented [3].

A final step of detailed design is needed to design transmissions, actuator placement and coupling systems if the hand is going to be underactuated. At this level, the design is usually done for the single finger, such as the designs of individual, underactuated fingers in [8] and [1] or finger configurations [17]. For the design in detail, most of the current efforts deal with what we can denote as planar fingers, that is, fingers for which all the joint axes are parallel, and also in some cases with perpendicular axes. In order to have functional hands with arbitrarily-oriented axes, different types of transmissions and coupling methods need to be explored.

2 Kinematic Synthesis of the Five-Fingered Multi-fingered Hand

The object of this work is to implement the design of a wristed, five-fingered hand which is a member of the single-jointed tree topology family studied in [5]. It has a single joint at the wrist and a single joint for each of the five fingers attached to the wrist with a single palm, see 1. It is solvable for $m_p = 2$ positions, basically a pick-and-release, or a single action task. Each finger has the same connectivity, and single mobility, plus the common mobility at the wrist.

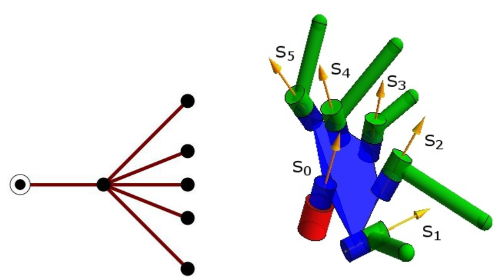

Fig. 1 The tree graph of the $1-(1,1,1,1,1)$ hand, left; the kinematic sketch, right.

The dimensional synthesis of this hand was studied in [7]. The complete description of the methodology for the synthesis can be found in [10], and it basically consists of solving the system of equations $\mathbf{F}(\mathsf{S},\Delta\theta)$, composed of the forward kinematics equations of each branch, equated to the relative displacement of the corresponding fingertip. For $b=5$ branches and a simultaneous task of $m_p=2$ finite positions per finger, we have the set of equations:

$$\mathbf{F}(\mathsf{S},\Delta\theta)=\hat{P}^c_{12}-e^{\frac{\Delta\theta_0}{2}\mathsf{S}_0}e^{\frac{\Delta\theta_c}{2}\mathsf{S}_c},\quad c=1,\ldots,5, \tag{1}$$

where $\hat{P}^c_{12}$ is the relative displacement from position 1 to position 2 for each finger c. The six joints have Plucker coordinates $\mathsf{S}_i=\mathbf{s}_i+\varepsilon\mathbf{s}_{i0}$, for $i=0,\ldots,5$, and the rotation angle about each joint is θ_i. The relative forward kinematics is computed as the product of exponentials for each brant. An algebraic derivation for this problem can be found in [7] and it yields two solutions.

The two positions shown in Table 1 for the first finger and in Figure 2 are selected. The solution from the synthesis problem yields the hand shown in Figure 2. In this sketch, the wrist is connected to the fingers through several rigid links forming the palm. The connection between the joints is drawn at the common perpendicular line between axes. The twist angles between the wrist joint and each finger joint are shown in Table 2.

Table 1 The two positions used for the synthesis of finger 1, expressed as dual quaternions

Finger	Position 1	Position 2
Finger 1	$0.94-0.19i-0.17j+0.20k+\varepsilon(-2.41-4.16i-3.69j+4.25k)$	$0.99-0.10i-0.04j+0.01k+\varepsilon(-0.76-6.76i-2.49j+0.50k)$

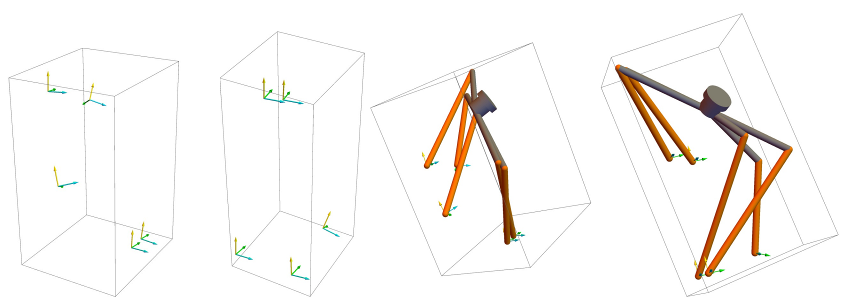

Fig. 2 First and second positions for each fingertip, left and schematic sketch of hand in both positions, right.

It is clear from the example that, even though the hand is theoretically capable of performing the task, its implementation is not straightforward.

Table 2 Twist angles α_{ij} between axis S_i and axis S_j.

Twist Angle	α_{01}	α_{02}	α_{03}	α_{04}	α_{05}
Value (degrees)	-161.7	-172.4	-175.7	-173.3	-145.7

3 Hand Optimization

The link-based optimization developed in [18] is used in order to select the best placement for the joints, that is, the dimensions and angles of the links. The method is based on optimizing the links' locations along each axes in order to satisfy a set of performance requirements.

Here, the objective function is formulated to minimize the overall length of the hand by considering the lengths between wrist joint and each of the five finger joints, as well as the length to the five end-effectors. The sliding scalar values along each of six axes to define the anchoring points are the parameters. Minimum and maximum link sizes were considered and formulated as non-linear constraints. Another important criterion is avoiding the self-intersection of hand parts. For this, links and end-effectors are considered as cylinders that must maintain a user-defined distance with respect to each other. These conditions create another set of nonlinear constraints. The formulation was made in Mathematica®and run in MATLAB®using *ga* and *fmincon*. The optimization took as average 15 generations and less than a minute. The resulting sliding parameters for each joint are shown in Table 3. These sliding values are measured from the intersection points at the common normal, so that their initial value is zero. Figure 3, shows the sketch of the hand before and and after optimization at position 1. It can be seen how all the link and finger lengths are more balanced after the optimization.

Table 3 Resulting values for the sliding parameters of the joints

ts	t_0	t_1	t_2	t_3	t_4	t_5
values	-13.796	-13.796	-12.698	-11.638	-5.608	10.438

An automatic modeling method developed in [3] was used on the optimization results to generate an initial model of the hand. Figure 4 presents the hand model in this stage at second position.

4 Underactuated Coupling Mechanisms

In order to reduce the complexity in the control and to allow for adaptive motion, different types of coupling mechanisms are used so that the hand can perform the

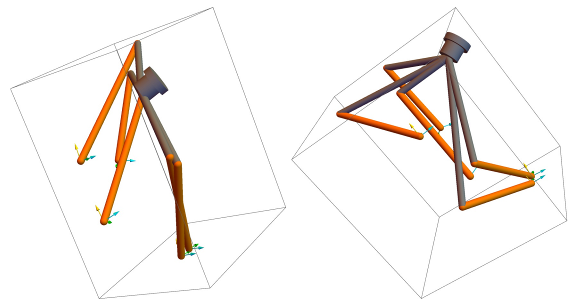

Fig. 3 Schematic sketches of the hand in position 1 before optimization, left and after optimization, right.

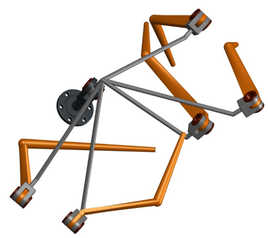

Fig. 4 Hand model created by automatic modeling in second position.

desired motion with a smaller number of actuators. For this design, a single actuator should perform the desired motion for all five fingers. Most of the mechanisms currently used are restricted to particular arrangements of the joint axes that are being coupled. The most common case is that of joints with parallel axes. The table below intends to be a non-exhaustive enumeration of the most popular coupling mechanisms and their applicability to different arrangements of the joint axes.

Table 4 Mechanisms to implement coupling between joint axes

Mechanism	Applicability
Spur Gears	Parallel joint axes
Helical Gears	Generally-oriented joint axes
Bevel Gears	Intersecting axes
Belts and Cables	Parallel axes
Closed-loop linkages	Generally-oriented axes

The use of some types of gears could be a good solution but must be complemented with another transmission such as a cable, because the distance between the axes is limited by the gear ratio and the size of the elements. This can be solved by using a combined gear + cable system.

Belts and cables present a similar problem; the force at the belt is perpendicular to the rotation axis for parallel axes, and those mainly tangential forces are optimal to create the desired torque and angular velocity. When the axes are skew, axial force components appear. The problem increases when the anchor points are not located at the common perpendicular line between axes. Those axial components increase the friction and require specific solutions to avoid the cable to slip from the pulley.

The use of additional links to create closed-loop linkages is a very common solution for parallel axes, because it yields a low-friction, low-maintenance and high-rigidity (if desired) coupling mechanism. The simplest way to connect two revolute joints with a linkage to yield a 1-dof system is the use of two more joints, in order to create a spatial four-bar linkage [16].

The simplest of the four-bar linkages consists of four revolute joints, and for skew joints the only movable four-bar linkage is the Bennett linkage. However, its use as coupling mechanism for an existing R-R chain is limited.

The synthesis of the five-fingered hand yields two solutions. Connecting both solutions yields a Bennett linkage for each finger. Figure 5 shows the Bennett linkages for the example in this paper, and it illustrates the limitations in the use of this method. Because there is only one option for coupling the hand, the designer has no control over the placement and dimensions of the coupling mechanism, which is not an acceptable solution in general.

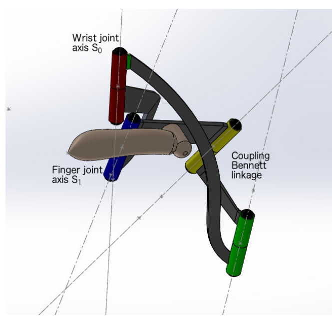

Fig. 5 The R-R chain for wrist joint and first finger, coupled using the complementary R-R chain to become a Bennett linkage.

Other spatial-four-bar linkages contain cylindric joints. Cylindric joints in general present problems of friction and linear motion range, yielding them also impractical as a coupling mechanism. Finally, the use of spherical joints in the design has several advantages. They are small and compact, and they yield non-overconstrained mechanisms, which increase the search space to optimize their placement and dimensions. One of the few drawbacks of this linkage is that the range of the spherical joints is limited.

5 The R-S-S-R Coupling Mechanism

The R-S-S-R mechanism is a non-overconstrained spatial four-bar linkage with mobility equal to one. This mechanism is regularly used for instance for function generation.

Each spherical joint can be seen, if no limits are imposed, as the rotation about a line whose axis that can take any direction, but must pass through the center of the sphere, C_i. This rotation can be expressed using dual quaternions as:

$$\hat{S}(\alpha) = \alpha_4 + \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{Bmatrix} + \varepsilon \begin{Bmatrix} c_x \\ c_y \\ c_z \end{Bmatrix} \times \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{Bmatrix}, \quad (2)$$

where $\mathbf{C}$ is the position vector for the center of the spherical joint and $\hat{\alpha} = \alpha_4 + \alpha_1\mathbf{i} + \alpha_2\mathbf{j} + \alpha_3\mathbf{k}$ contains the rotation components.

The S-S chain can be synthesized for up to seven positions. This means that a design to reach the two positions of each finger has five free parameters and a single constraint in the location of the spherical joint centers, $\mathbf{C}_1$ and $\mathbf{C}_2$. This condition can be found from the synthesis equation for the chain. If we pre- or post-multiply by one of the joint rotations, we obtain a set of equations that are linear and homogeneous in the axis-angle of rotation variables,

$$\hat{S}_2(\beta) = \hat{S}_1^*(\alpha)\hat{P}_{12}, \quad (3)$$

where $\hat{P}_{12} = p_0 + p_1 i + p_2 j + p_3 k + \varepsilon(p_4 i + p_5 j + p_6 k)$ is the relative motion between position 1 and position 2 expressed also in dual quaternions, and $\hat{S}_1^*(\alpha)$ is the conjugate displacement for the first S joint.

We can use a standard elimination process to write the inverse kinematics solution for the axes-angles α and β, which yields the 8×8 matrix system

$$\begin{bmatrix} p_0 & p_3 & -p_2 & -p_1 & 1 & 0 & 0 & 0 \\ -p_3 & p_0 & p_1 & -p_2 & 0 & 1 & 0 & 0 \\ p_2 & -p_1 & p_0 & -p_3 & 0 & 0 & 1 & 0 \\ -p_1 & -p_2 & -p_3 & -p_0 & 0 & 0 & 0 & 1 \\ c_{1y}p_2 + c_{1z}p_3 + p_7 & -c_{1z}p_0 - c_{1x}p_2 + p_6 & c_{1y}p_0 - c_{1x}p_3 - p_5 & -p_4 & 0 & -c_{2z} & c_{2y} & 0 \\ c_{1z}p_0 - c_{1y}p_1 - p_6 & c_{1x}p_1 + c_{1z}p_3 + p_7 & -c_{1x}p_0 - c_{1y}p_3 + p_4 & -p_5 & c_{2z} & 0 & -c_{2x} & 0 \\ -c_{1y}p_0 - c_{1z}p_1 + p_5 & c_{1x}p_0 - c_{1z}p_2 - p_4 & c_{1x}p_1 + c_{1y}p_2 + p_7 & -p_6 & -c_{2y} & c_{2x} & 0 & 0 \\ -c_{1z}p_2 + c_{1y}p_3 - p_4 & c_{1z}p_1 - c_{1x}p_3 - p_5 & -c_{1y}p_1 + c_{1x}p_2 - p_6 & -p_7 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{Bmatrix} = \mathbf{0} \quad (4)$$

The condition on the joint centers is stated as imposing that the matrix must have rank lower than eight in order to have a nonzero solution for the rotation variables. This yields a quartic condition on the parameters of the spherical joint centers, $\mathbf{C}_1$ and $\mathbf{C}_2$.

Exact synthesis can be performed by selecting six of the seven parameters and solving for the remaining one. Instead of the exact synthesis problem, in this case we convert it to an optimization problem for locating the transmission joints with respect to the hand in order for the designer to add some control on requirements.

A similar approach to the hand optimization was followed. Here, the objective function minimizes the length of S-S chain of each finger separately, having the sphere centers $\mathbf{C}_1$ and $\mathbf{C}_2$ positions as parameters. The limits for the location of each R-S chain with respect to the hand are formulated as nonlinear constraints. Also, the distance between $\mathbf{C}_1$ and $\mathbf{C}_2$ was limited to be similar to the corresponding distance along the R-R chain. In addition, the S-S chain reachability to the positions was also added to the minimization problem as an exact nonlinear constraint.

Table 5 presents the optimization results.

Table 5 Optimized positions for first S-S. Point coordinates with respect to hand fixed frame.

Joint Centers	x	y	z
$\mathbf{C}_1$	-19.867	-6.314	13.709
$\mathbf{C}_2$	-14.330	4.298	21.352

6 Final resulting mechanism

The method has been applied here for Finger 1. In Figure 6 it can be seen how the S-S coupling mechanism is compact and after the optimization, does not intersect any of the links for the motion of the finger.

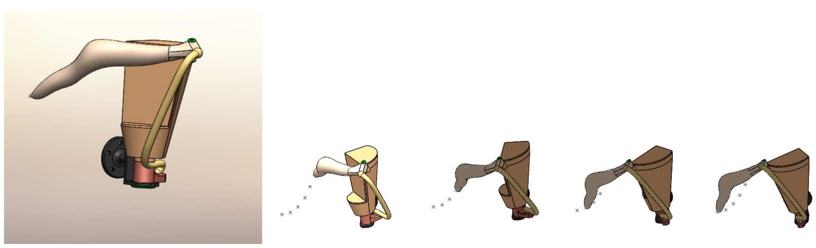

Fig. 6 The final R-S-S-R mechanism for Finger 1.

7 Conclusions

The present work is an attempt to further investigate the implementation of non-standard robotic hands designed for specific tasks. These hands appear when directly applying synthesis procedures in order to create robotic hands for simultaneous tasks of all fingertips. In mot of the cases, they present skew axes that increase the complexity when designing detailed parts and transmissions. In particular, a linkage-based transmission for underactuated hands with skew axes is studied. This consists of coupling the R-R joint corresponding to wrist joint and finger joint with an S-S mechanism, creating a 1-dof system that can be actuated at the joint. The R-S-S-R linkage allows enough flexibility to be able to state the design problem as an optimization problem to obtain optimum values for length, placement and obstacle avoidance.

Acknowledgements This work is supported by the National Science Foundation under Grant No. 1208385. The content is solely the author's responsibility.

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