Singularity Design for RRSS Mechanisms

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Abstract This paper explores the design of RRSS mechanisms with specified singular positions. The ultimate goal is to find an SS chain to couple a previously synthesized RR chain and to force the mechanism to become singular at intended positions. To this end, a system of equations including the closure equation and its derivative with respect to the joint values as well as a constraint imposing the prior design poses are utilized. This system acts as constraints for the optimization of an objective function to minimized the lengths of the links. The solution yields an SS chain that allows the mechanism to go through the task positions and to be singular at the selected positions.

1 Introduction

The characterization of singularities for closed-loop or parallel mechanisms has been widely studied. Single-loop linkages were the first ones to be studied and are the focus of this work. Among many works, we highlight [4], where the mobility problem for four-bar linkages was studied by finding the global extrema of a quadratic function on a cylinder and solving it as the geometric problem of the intersections between a circle and a hyperbola. Later, [1] described three possible types of singularities based on Jacobian matrices for the loop equations. More recently, [7] shows that the

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singularities of the configuration space and degeneracy of the kinematics of a chain are not always related to each other. Focusing on the RSSR linkage, [2] analyzed the first type of singularity.

Works involving design include [5], where the general reason for losing rank and find ill-condition in synthesis was analyzed and solved with an optimization procedure. In [8], Grashof conditions were discussed for closed-loop RSSR mechanisms, and the use of a geometrical approximation to find conditions that make crank-rocker mechanisms. Most of the works found focus on avoiding singularities or creating singularity-free trajectories such as [6], with an algorithm based on a Probabilistic RoadMap that can generate trajectories avoiding singularities.

Our interest started with [3], in which the intention was to use SS chains to couple RR fingers for the design of wristed, multi-fingered robotic hands. Unlike previous approaches, in this work, the focus is on designing the mechanism for specific singular configurations at desired positions. This approach can be useful to make sure the designed mechanism will stop at specific locations, or have motion between two specific points, corresponding to the two singular points. In the following sections, the derived equations are explained and examples are included to test the method.

2 The RRSS Mechanism

In designs for which the robot size is important and the mechanism has to work in a small space, the required number of actuators plays an important role in minimizing the final size of the robot. Therefore, the smaller the number of actuators, the more control the designer has over the size of the robot to target smaller dimensions. In the RRSS mechanism, by coupling the RR chain with two S joints, the degrees of freedom of system is decreased to one. This helps to minimize the number of required actuators for the mechanism. A similar mechanism is discussed in [3] with five fingers and just 1 degree of freedom for all the fingers. Figure 1 shows this mechanism with 5 RRSS closed linkages, which is used for a robotic hand.

In the present work, for designing an RRSS mechanism, the first step is to synthesize an RR chain based on the design objective, mainly the positions that the system must reach. More details on finding the RR chain for a specific task is explained in [3].

A spherical joint S can be expressed using dual quaternions as:

$$\hat{S}(\alpha) = \alpha_0 + \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{Bmatrix} + \epsilon \begin{Bmatrix} c_x \\ c_y \\ c_z \end{Bmatrix} \times \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{Bmatrix} \tag{1}$$

where the position of the center point of the joint is $c = [c_x, c_y, c_z]$ and α contains the rotation components. When the SS chain is attached to the RR chain to form the RRSS linkage, the SS will have the same relative displacement as the RR chain. Consider the relative motion in dual quaternion form $\hat{P_{12}} = P_0 + P_1 i + P_2 j + P_3 k + \epsilon(P_4 i + P_5 j + P_6 k + P_7)$. Relation between S joints and relative motion shows in

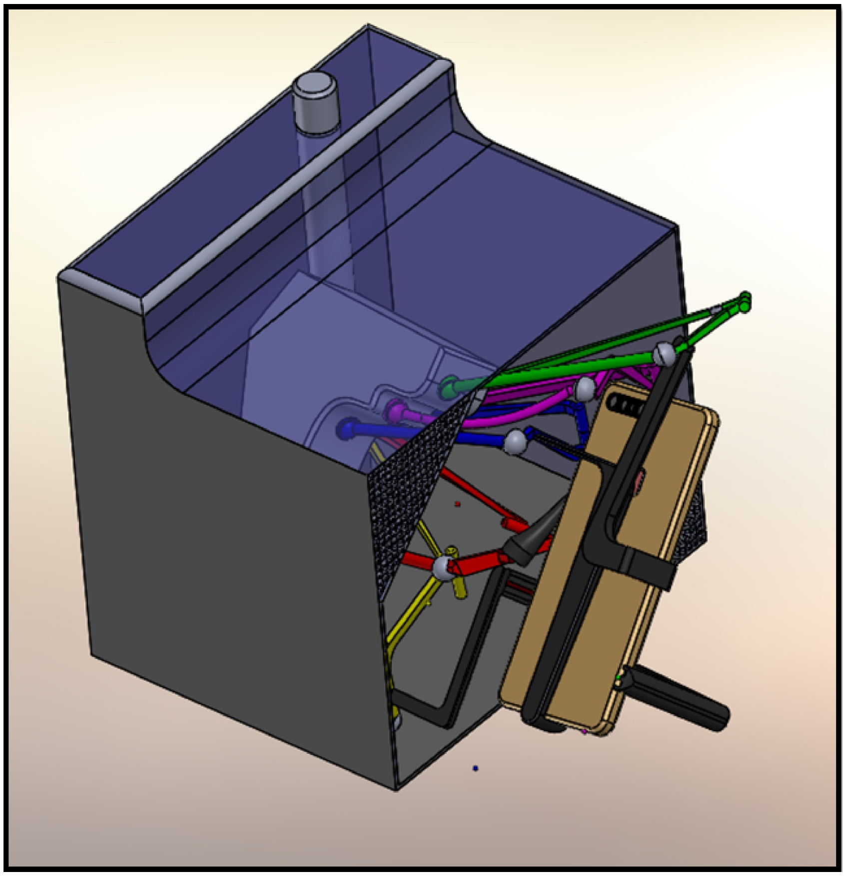

Fig. 1 Mechanism with 5 RRSS chains.

equation 2. In this equation (α) and (β) are rotation components belong to first and second S joint respectively.

$$\hat{S}_2(\beta) = \hat{S}_1^*(\alpha)\hat{P}_{12} \tag{2}$$

Finally, we impose that the SS chain goes to the desired positions in Equation 3. In this equation, the determinant of the 8×8 matrix M has to be zero (not a full rank). Thus, Equation 4 would be a condition for the S joints in the RRSS mechanism.

$$\begin{bmatrix} P_0 & P_3 & -P_2 & -P_1 & 1 & 0 & 0 & 0 \\ -P_3 & P_0 & P_1 & -P_2 & 0 & 1 & 0 & 0 \\ P_2 & -P_1 & P_0 & -P_3 & 0 & 0 & 1 & 0 \\ -P_1 & -P_2 & -P_3 & -P_0 & 0 & 0 & 0 & 1 \\ C_{1y}P_2 + C_{1z}P_3 + P_7 & -C_{1z}P_0 - C_{1x}P_2 + P_6 & C_{1y}P_0 - C_{1x}P_3 + P_5 & -P_4 & 0 & -C_{2x} & C_{2y} & 0 \\ C_{1z}P_0 - C_{1y}P_1 - P_6 & C_{1x}P_1 + C_{1z}P_3 + P_7 & -C_{1x}P_0 - C_{1y}P_3 + P_4 & -P_5 & C_{2x} & 0 & -C_{2x} & 0 \\ -C_{1y}P_0 - C_{1z}P_1 + P_5 & C_{1x}P_0 - C_{1z}P_2 - P_4 & C_{1x}P_1 + C_{1y}P_2 + P_7 & -P_6 & -C_{2y} & C_{2x} & 0 & 0 \\ -C_{1z}P_2 + C_{1y}P_3 - P_4 & C_{1z}P_1 - C_{1x}P_3 - P_5 & -C_{1y}P_1 + C_{1x}P_2 - P_6 & -P_7 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{Bmatrix} = 0 \quad (3)$$

$$det[M] = 0 \quad (4)$$

3 Singularity Design Equations for the RRSS Mechanism

Any closed-loop chain can be characterized as having input and output joint angles, which are represented with n-dimensional vectors θ_{in} and i-dimensional vectors θ_{out} respectively. These input and output vectors are related to each other according to the overall number of degrees of freedom of the system.

One of the methods to characterize singularities [1] is using loop closure equations. This is shown in Equation 5.

$$\left[J\right]\left[\dot{\theta}_{in}\right] = \left[J^*\right]\left[\dot{\theta}_{out}\right] \quad (5)$$

In general, there are three types of singularities, which can occur separately or simultaneously [1].

For the first type of singularity,

$$\left[\dot{\theta}_{in}\right] = 0, \qquad \left[\dot{\theta}_{out}\right] \neq 0, \quad (6)$$

Therefore:

$$det[J^*] = 0. \quad (7)$$

The second type of singularity happens when

$$[\dot{\theta}_{out}] = 0, \quad [\dot{\theta}_{in}] \neq 0. \quad (8)$$

Therefore:

$$det\left[J\right] = 0 \quad (9)$$

Finally, the third type of singularity is a combination of first and second types of singularity. In other words, $det[J] = det[J^*] = 0$ or both matrices J and J^* become singular at the same time [1]. The present work focuses on an RRSS linkage, a single-loop mechanism. The RRSS linkage has two degrees of freedom, one being the idle rotation about the link connecting the two spherical joints. Figure 2, shows a single RRSS closed chain.

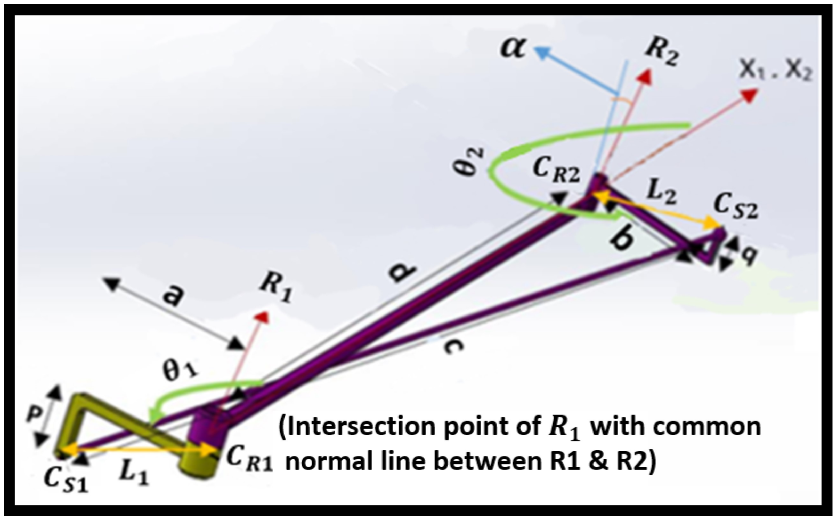

Fig. 2 Single RRSS chain

As it was stated before, the RR chain is designed a priori for a given task and then coupled with the SS. That means that RR parameters (Plucker coordinates ($R1$,$R2$), twist angle α, distance between R joints d) are known. However, locations of S joints and input/output angles (θ_1, θ_2) are unknown. To calculate the closure equation for this mechanism, we create a distance constraint by taking the two paths originating from joint R_1. Equation 10 shows the transformation needed to reach C_{S1} (center point of first S joint) from R_1 in the global coordinate. In following equation R and T represent rotation and translation along common normal lines in homogeneous format respectively.

$$C_{S1} = [G][R_{\theta 1}][T_a][T_p]\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix} \tag{10}$$

in which $[G]$ is a transformation from the first revolute joint (R_1) coordinate, where we considered our local frame origin, to the global coordinate. The same procedure applies for the right leg starting from R_1 and leading to location of point C_{S2} (center point of second S joint):

$$C_{S2} = [G][T_d][R_\alpha][R_{\theta 2}][T_b][T_q]\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix} \tag{11}$$

Having the coordinates of the two spherical joints in terms of the structural parameters and joint variables of the mechanism, the final loop closure can be formulated as Equation 12, where c^2 is the square of the length of the link between the two spherical joints, which can be computed by the difference between location of the center point of S joints from the left and right legs, Equations 10 and 11.

$$F = (C_{S1} - C_{S2}) \cdot (C_{S1} - C_{S2}) - c^2 = 0 \tag{12}$$

Therefore, loop closure in Equation 13 is computed as a function of (a, b, c, p, q) which are link lengths and the input and output angles (θ_1 and θ_2). Singularity equations for type 1 and type 2 can be found from Equations 14 and 15 where the output and input velocity is not zero respectively. Any solution to (Equation 13) and one of Equations 14 or 15 forces the mechanism to be singular at a particular point.

$$F(\theta_1, \theta_2, a, b, p, q, c) = 0 \tag{13}$$

$$\frac{\partial F}{\partial \theta_2} = 0 \tag{14}$$

$$\frac{\partial F}{\partial \theta_1} = 0 \tag{15}$$

For each particular value for θ_1, we create a singular pose for which two equations need to be satisfied. We can keep adding these singular points up to the maximum number of singularities for the linkage. In the present work, the equations are solved for two singular poses for which we can write the following set of equations:

$$\begin{aligned} \theta_{11} => &\begin{cases} F(\theta_{21}, a, b, p, q, c) = 0 \\ \frac{\partial F}{\partial \theta_1}(\theta_{21}, a, b, p, q, c) = 0 \end{cases} \\ \theta_{12} => &\begin{cases} F(\theta_{22}, a, b, p, q, c) = 0 \\ \frac{\partial F}{\partial \theta_1}(\theta_{22}, a, b, p, q, c) = 0 \end{cases} \end{aligned} \tag{16}$$

where θ_{11} and θ_{12} are the given values of the first joint (θ_1) and θ_{21} and θ_{22} are the unknown variables of the second joint (θ_2).

Adding Equation 4 to 16 provides a system of equations whose solution is an RRSS mechanism with two specified singular poses and a workspace which includes the poses that the RR chain was synthesized for. We transform the design into an optimization problem whose objective function, G, is the sum of the squares of the lengths of all links (except "d" as it was calculated in RR synthesis step) to have some control over the total size of the mechanism. Equation 17 shows this objective function. This objective function was chosen to design a mechanism with the most appropriate size.

$$G = a^2 + b^2 + p^2 + q^2 + c^2 \tag{17}$$

4 Solution Procedure and a Numerical Example

For designing a single RRSS chain and to show that the proposed method can be applied to any arbitrary RRSS, we start with selecting two random task positions and find the appropriate RR chain for these points from synthesis. Two task positions and two points on the axis of R joints are included in Table 1. Also, the Plucker coordinates for each of the R joints are shown in Table 2. Then we can couple this RR chain with an SS linkage under the conditions mentioned in Equation 4 for S joints.

Table 1 Two task positions and two points on each R joint axis

Parameters	X (cm)	Y (cm)	Z (cm)
Task Position 1	5.2	-7	-3
Task Position 2	0	-3.75	0.4
Axis of R Joint 1-Point 1	-7.02	20.73	-1.63
Axis of R Joint 1-Point 2	-7.65	20.49	-2.24
Axis of R Joint 2-Point 1	-8.07	-0.10	-0.36
Axis of R Joint 2-Point 2	-10.46	4.38	6.85

Table 2 Plucker coordinates for the R joints

Joints	S	S^0
1st R Joint	-0.70, -0.26, -0.67	-14.32, -3.57, 16.23
2nd R Joint	-0.27, 0.51, 0.82	0.095, 6.69, -4.13

For this design with two specific singular positions, we have five equations (four of them from Equation 16 plus one main condition for S joints from Equation 4), and seven unknown design parameters (a, b, c, p, q, θ_{21}, θ_{22}). All five equations are nonlinear and they will be added as equality constraints to an optimization problem. The two arbitrary input angles (θ_{11}, θ_{12}), where we want the mechanism to be singular, are selected and presented in Table 3. For solving the optimization problem we use a genetic algorithm (ga) with an upper limit of 400 generations, Constraint Tolerance (TolCon) of $1e-6$. The objective function was defined as minimizing the sum of squares of the length of each link.

The optimization test was run 20 times and the best result based on the minimum objective function, time of the test, and number of generations was selected. In the best result, the optimization finished after 200 generations and a total time of 170.92 s. The final value for the objective function was 454.208 cm. After finding unknown parameters, the center point for each of S joints can be found. Table 4 expresses the results for all unknown parameters and the center point of S joints.

The final design of the model is shown in Fig. 3. In this case the finger (RR chain) is designed to move from point 1 to point 2. These two points are objective design values, which are defined for RR chain synthesis. Also, two selected singularity

Table 3 Information about relation between R joints and input values for θ_1 to define two singular positions

Parameters	Value (cm) or (rad)
Length of link d	18.71
Angle α	1.06
Angle θ_{11}	0.39
Angle θ_{12}	1.05

Table 4 Result for unknown parameters and center point of S joints for RRSS with two specified singular positions

Parameters	Value (cm) or (rad)
Length of link a	14.92
Length of link b	-7.70
Length of link p	1.81
Length of link q	-10.05
Length of link c	6.06
θ_{21}	5.55
θ_{22}	5.67
1st S Joint	-4.45, 6.58, 2.47
2nd S Joint	-2.01, 4.28, -2.57

points for this robot are in $\theta_{11} = \pi/8$ and $\theta_{12} = \pi/3$, which would occur at points 1 and 3. Figures 4 and 5 show the angles for these two singularity points.

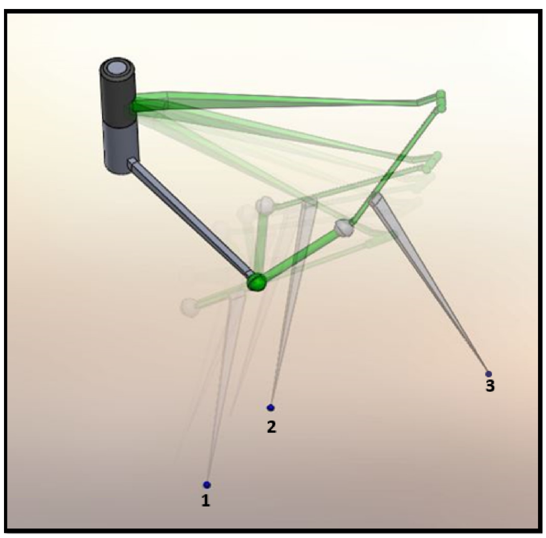

Fig. 3 Motion of RRSS chain between points 1 and 2 (the points the RR chain was synthesized for) plus singular positions at point 1 and 3.

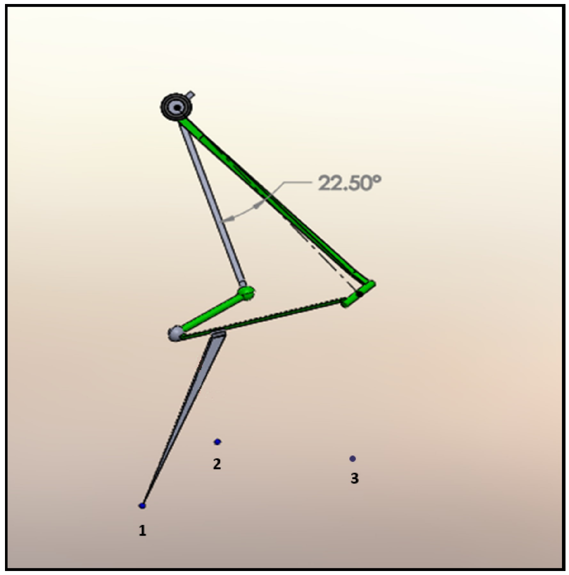

Fig. 4 Angle for θ_{11} at the first singularity position (top view)

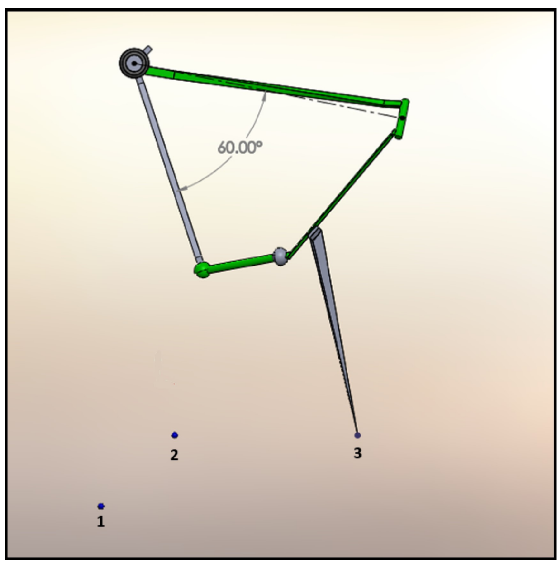

Fig. 5 Angle for θ_{12} at the second singularity position (top view)

The velocity of the output angle can be found from $\frac{\partial F}{\partial \theta_2} \frac{\partial \theta_2}{\partial t}$. In the points where the sign of the velocity changes (and velocity become zero), the mechanism becomes singular. Therefore, if we sketch the graph for $\frac{\partial F}{\partial \theta_2}$ respect to θ_2 for any specific θ_1, we can find singular points for this mechanism.

Figure 6 shows the results for the example above. From this figure, there are two possible singular points for each selected θ_1. Also, the relation between θ_1 and θ_2 for loop closure is presented in Figure 7, which shows just one of these singular points from Figure 6 for each θ_1 belonging to the workspace of the robot. This coincides with the point selected to be singular. These values are mentioned in Table 5.

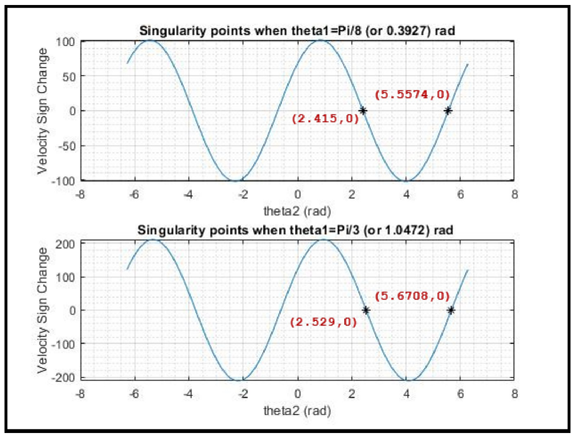

Fig. 6 Possible singular points for each selected θ_1

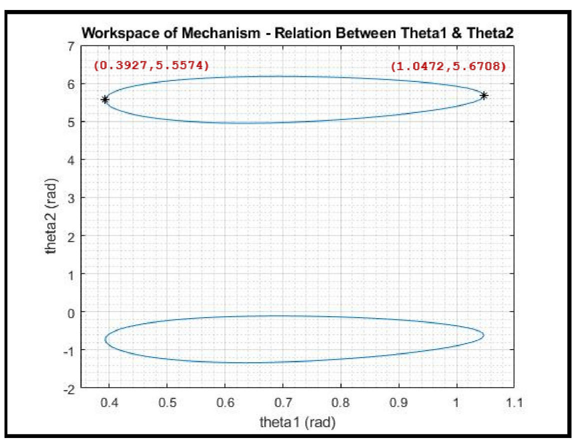

Fig. 7 The relation between θ_1 and θ_2 for the loop closure

Table 5 Possible singularity points and valid one respect to work-space of robot

θ_1	θ_2	Description	Validity
0.39	2.41	Extra possible singular point	Out of Robot Workspace
0.39	5.56	Singular point defined by designer	Inside of Robot Workspace
1.05	2.52	Extra possible singular point	Out of Robot Workspace
1.05	5.67	Singular point defined by designer	Inside of Robot Workspace

5 Conclusion

In this work, an RRSS closed linkage with known singular poses was designed by coupling an SS chain to an already-synthesized RR chain. A constraint equation and the singularity condition of type one have been utilized to specify singular poses. These equations are populated by new values for the input angle to create a system of design equations whose solution is a mechanism with desired singular poses. The method has been validated by a numerical example and simulate. Future work will explore generalizing the concept for other closed linkages.

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