Discussion of Objectives

Unit 1. Linear Equations

Objective 1a. Solve formulas, and use formulas to solve applied problems.

OpenStax sections and exercises

2.3, Solve a Formula for a Specific Variable, odd as needed 165 - 237
7.4, Solve Rational Equations, odd as needed 235 - 249

A formula is an equation that describes a general relation between two or more quantities. For Example, the formula $P = 2\ell + 2w$ relates the perimeter $P$ of a rectangle to the length $\ell$ and the width $w$.

Typical questions. You should be able to solve a given formula for a specified variable—that is, rewrite the formula so that one side consists of the variable by itself and the other does not involve the variable. Given numerical values for all but one of the variables, you should also be able to determine the value of the remaining variable.

Example. Solve the formula $P = 2\ell + 2w$ for $w$.
Solution: $w = (P - 2\ell)/2$

Example. Use the formula $P = 2\ell + 2w$ to find the width of a rectangle whose perimeter is 42 inches and whose length is 12 inches.
Solution: 9 inches

Objective 1b. Solve absolute-value equations, and express the solution in set and graphical notation.

OpenStax section and exercises

2.7, Solve Absolute Value Inequalities, odd as needed 435 - 455

The objective here is to solve absolute value equations such as $|x + 2| = 5$. Absolute value equations can have one, two or no solutions. To solve the equation means to describe the set of solutions, and there are three ways to do that: algebraically reduce the equation to specific conditions such as $x = -7, x = 3$; use set notation such as $\{-7, 3\}$; or graph the solution set on the real number line.

Typical questions. Given an absolute-value equation, you should be able to describe the solution set in set notation and graphically.

Example. Solve $|x - 3| = 10$ algebraically. Then give the solution set using set notation, and graph it.
Solution: $x = -7$ or $x = 13$; $\{-7, 13\}$; graph omitted

Example. Solve $-3x + 5 = 7$.
Solution: $x = -2/3$ or $x = 4$

Example. Solve $| -3x - 5| = |2x + 2|$.
Solution: $x = -7/5$ or $x = -3$
Objective 1c. Graph linear equations in two variables by finding and plotting the intercepts.

OpenStax section and exercises
3.1, Graph Linear Equations in Two Variables, odd as needed 9 - 27, 33 - 67

In the \(xy\)-plane, a straight line with defined and nonzero slope necessarily has both an \(x\)- and a \(y\)-intercept. Every \(y\)-intercept has an \(x\) coordinate equal to zero, and every \(x\)-intercept has a \(y\) coordinate equal to zero. N.B. Every vertical line in the \(xy\)-plane has undefined slope and an \(x\)-intercept, and every horizontal line in the \(xy\)-plane has zero slope and a \(y\)-intercept.

Typical questions. Given an equation of a line, you should be able to graph the line by finding and plotting the \(x\)- and \(y\)-intercepts expressed as ordered pairs.

Example. Find the \(x\)- and \(y\)-intercepts of \(2x + 3y = 6\) expressed as ordered pairs.
Solution: \(x\)-intercept : \((3, 0)\); \(y\)-intercept : \((0, 2)\); plot and graph omitted

Example. Find the \(x\)- and \(y\)-intercepts of \(\frac{4}{3}x - 8y = -6\) expressed as ordered pairs.
Solution: \(x\)-intercept : \((-\frac{9}{2}, 0)\); \(y\)-intercept : \((0, \frac{3}{4})\); plot and graph omitted

Objective 1d. Determine the slope and intercepts of the line defined by a linear equation in standard, slope-intercept, or point-slope form, and graph the line.

OpenStax section and exercises
3.2, Slope of a Line, odd as needed 81, 83, 93 - 115, 125-149

The objective here is to obtain information about a line, such as its slope and intercepts, from an equation for the line and use that information to graph it. The equation might be given in standard form \(Ax + By = C\), slope-intercept form \(y = mx + b\), or point-slope form \(y - y_1 = m(x - x_1)\). An equation of the form \(y = c\) describes a horizontal line, and one of the form \(x = c\) describes a vertical line. Parallel lines have the same slope, and the slopes of perpendicular lines are negative reciprocals of each other.

Typical questions. You should be able to tell when a given linear equation describes a horizontal or vertical line. More generally, given a linear equation in any form, you should be able to determine the slope of the line that the equation describes and determine the \(x\)- and \(y\)-intercepts. Finally, you should be able to determine whether the lines described by two given equations are parallel, perpendicular, or neither by computing their slopes.

Example. Determine the slope and intercepts of the line \(y = 3x - 6\), and graph it.
Solution: Slope 3; \(y\)-intercept \((0, -6)\); \(x\)-intercept \((2, 0)\); graph omitted

Example. Determine the slope and intercepts of the line \(x = 3\), and graph it.
Solution: Undefined slope (a vertical line); no \(y\)-intercept; \(x\)-intercept \((3, 0)\); graph omitted

Example. Determine whether the lines \(2x + y = 4\) and \(y = 3x + 1\) are parallel, perpendicular or neither.
Solution: Slopes are \(-2\) and \(3\), respectively, so neither parallel nor perpendicular.
Example. Determine the slope and intercepts of the line $3x + 4y = 12$.
Solution: Slope $-3/4$; $y$-intercept $(0, 3)$; $x$-intercept $(4, 0)$

Example. Determine whether the lines $y = -4x + 7$ and $y = (1/4)x - 3$ are parallel, perpendicular or neither.
Solution: Slopes are $-4$ and $1/4$, respectively, so are perpendicular.

Objective 1e. Compute the slope of a line given two points, and find any form of the equation of a line given two points or the slope and one point.

OpenStax sections and exercises
3.2. Slope of a Line, odd as needed 73-79, 85-91
3.3. Find the Equation of a Line, odd as needed 155 - 233

This objective is the reverse of Objective 1d; it is to obtain an equation for a line given information that determines the line.

Typical questions. You should be able to rewrite a given equation for a line in another form, such as slope-intercept form. In other questions that ask for an equation of a line, at least one point on the line will be given. (Note: To know an intercept is to know a point.) The key to obtaining an equation is then to find the slope. That might be given, or you might need to compute it from other information, such as a second point that is on the line or a statement that the line is parallel or perpendicular to some other given line. Having the slope, you can immediately write down a point-slope equation $y - y_1 = m(x - x_1)$ and rearrange if necessary to obtain slope-intercept or standard form.

Example. Find the slope-intercept equation of the line $2x - 5y = 7$.
Solution: $y = \frac{2}{5}x - \frac{7}{5}$

Example. Find the slope of the line through $(1, 2)$ and $(4, -5)$.
Solution. $-7/3$

Example. Find a standard-form equation for the line through $(3, -1)$ with slope $1/2$.
Solution: $x - 2y = 5$ (other solutions possible)

Example. Find an equation for the horizontal line through $(−2, 5)$.
Solution: $y = 5$

Example. Find a point-slope equation for the line through $(1, 3)$ that is perpendicular to the line $y = 5x - 2$.
Solution: $y - 3 = -\frac{1}{5}(x - 1)$