Fluctuating Hydrodynamics, as originally devised by Landau and Lifshitz [1], offers a natural way for extending Navier-Stokes equations through the inclusion of a stochastic stress tensor. The latter represents the coarse-grained (in time and space) effect of the thermal agitation of the fluid molecules, generating local fluctuations of density and momentum. Such a perspective has been providing important insights into homogenous and heterogeneous nucleation dynamics which have been proved crucial for fundamental aspects, such as the extension of Classical Nucleation Theory to the inclusion of the surface tension curvature corrections [2a, 2b], corrections to the dissipation spectra in turbulence [3] and anomalous diffusion [4], as well as promising more realistic modelling of nucleation in fuel cells, cavitation in propellers and instabilities in injectors.

In this talk I will present some recent developments [5] concerning Fluctuating Hydrodynamics in the context of the mesoscopic multi-phase lattice Boltzmann method, which is a kinetic-based model for simulating fluid flows. I will show how, in the case of the Shan-Chen multi-phase model, it is possible to match with very high accuracy the two-point correlation function for the density fluctuations, i.e. the static structure factor. The exact implementation of momentum conservation at the lattice level allows for an unprecedented matching between theory and simulations, while opening the possibility to finely tune the structure factor at all wave-lengths recovering fundamental experimental features typically absent in previous models. These results pave the way for a coarse-graining scheme reliably bridging the microscopic physics and the mesoscopic modelling.

[2b: ML, L. Biferale, G. Falcucci, M. Sbragaglia and X. Shan. PRE 105, 015301 (2022)]