Name: ________________________________

Math 1143 Final Exam
December 12, 2011

Instructions. Be sure to show your work! A correct solution with no supporting work might receive no credit. Give numerical answers in exact form, such as $5/7$ or $3\sqrt{5}$, unless a problem asks for a decimal approximation. There are 100 points altogether.

Score:

1.	____	16.	____
2.	____	17.	____
3.	____	18.	____
4.	____	19.	____
5.	____	20.	____
6.	____	21.	____
7.	____	22.	____
8.	____	23.	____
9.	____	24.	____
10.	____	25.	____
11.	____	26.	____
12.	____	27.	____
13.	____	28.	____
14.	____	29.	____
15.	____	30.	____

Total: ______

(2 pts) 1. Determine the domain of the function.

(a) $y = \sqrt{x+3}$

(b) $y = \dfrac{x-2}{x+4}$

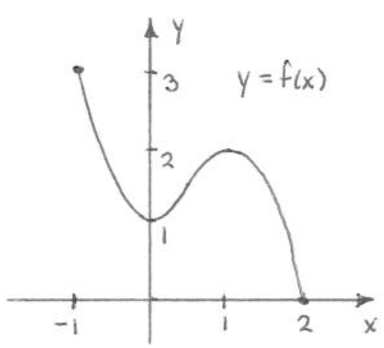
(3 pts) 2. Let $f(x) = x^2 - 2$. Compute the following, and simplify your answers:

(a) $f(-3)$

(b) $\frac{1}{2}f(2x)$

(c) $[f(x)]^2$

(4 pts) 3. For the function f graphed below:



(a) What is the domain?

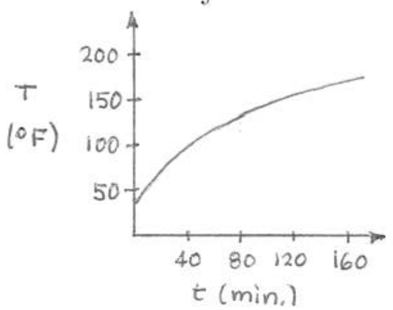
(b) What is the range?

(c) What is the maximum value?

(d) Which is larger, $f(1)$ or $f(2)$?

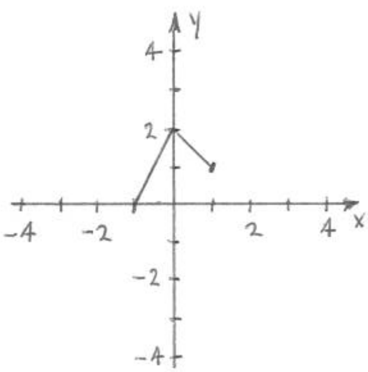
(3 pts) 4. A turkey is put into a hot oven. Its temperature t minutes later, in °F, is given by $T(t) = (250t + 4000)/(t + 100)$.

(a) Determine the average rate of change $\Delta T/\Delta t$ over the interval $0 \leq t \leq 40$. Include units in your answer.

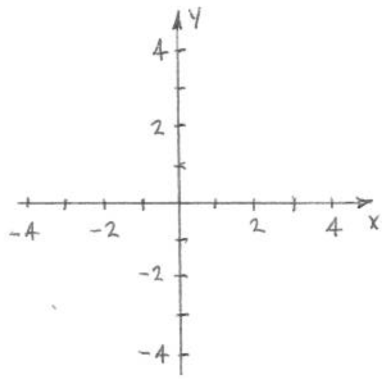


(b) Is the average rate of change over the interval $40 \leq t \leq 80$ larger or smaller than the average rate of change over the interval $0 \leq t \leq 40$? No explanation necessary.

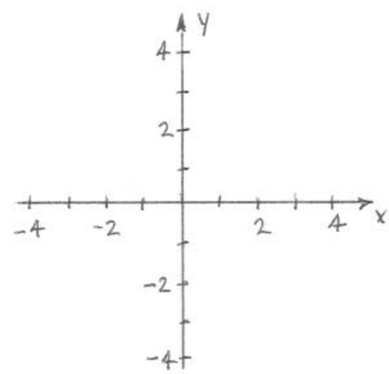
(4 pts) 5. Given the graph $y = f(x)$, sketch the graphs $y = f(x - 2) + 1$ and $y = -f(x)$.



$y = f(x)$



$y = f(x - 2) + 1$

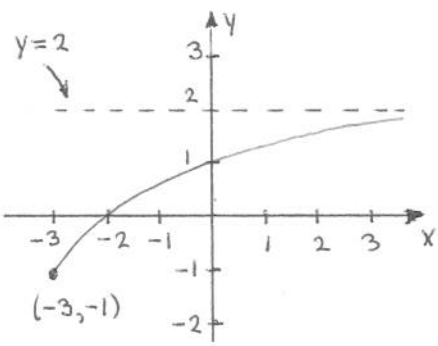


$y = -f(x)$

(2 pts) 6. Find $(f \circ g)(5)$ if $f(x) = x^2$ and $g(x) = \dfrac{x+1}{x-2}$.

(2 pts) 7. Express $h(x) = \sqrt{3x+4}$ as a composite $h = f \circ g$ of simpler functions f and g.

(3 pts) 8. A graph $y = f(x)$ is given below. The graph extends infinitely far to the right.



Determine $f^{-1}(0)$.

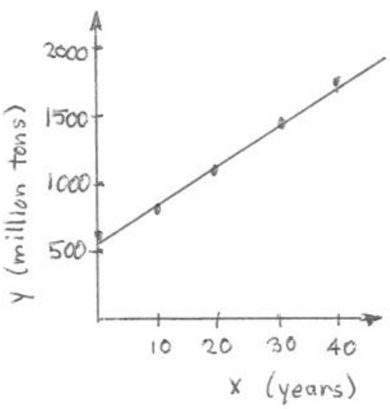
What is the domain of f^{-1}?

Add the graph $y = f^{-1}(x)$ to the sketch.

(3 pts) 9. As in Problem 4, suppose that the formula $T(t) = (250t + 4000)/(t + 100)$ gives the temperature in °F of a baking turkey t minutes after it was put into an oven. Given the formula $T^{-1}(x) = (100x - 4000)/(250 - x)$, find $T^{-1}(180)$, and explain what that number tells you in real-life terms. (Note: Do not *find* $T^{-1}(x)$; just use the formula and interpret your answer.)

(3 pts) 10. A manufacturer buys a machine for \$80,000. Its expected lifetime is 10 years, at which point it will have a salvage value of \$8000. Assuming linear depreciation, give a formula for the value $V(t)$ after t years, where $0 \leq t \leq 10$.

(4 pts) 11. The scatter plot below portrays annual worldwide grain production, in millions of tons, since 1950. The regression line for the data is $y = 28.8x + 570$, where x is the number of years since 1950. Use the regression line to predict the following, rounded to the nearest whole number:



Grain production in 2011 (in millions of tons).

Year in which production will reach 2500 million tons.

(4 pts) 12. For the parabola $y = 4(x - \frac{1}{2})^2 - 2$, determine:

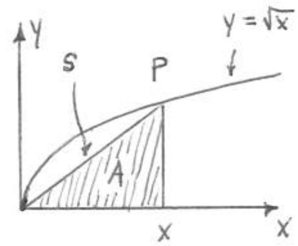
(a) The vertex

(b) The axis of symmetry (Note: This is a line, not a number.)

(c) The y-intercept

(d) The x-intercepts in exact form

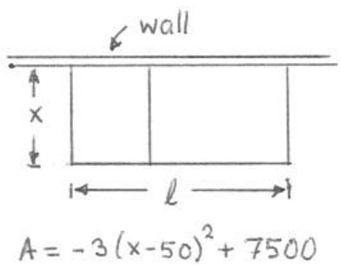
(4 pts) 13. If $P(x, y)$ is on the curve $y = \sqrt{x}$, express the distance s from the origin to P and the area A of the shaded triangle as functions of x. Show your work.



s as a function of x:

A as a function of x:

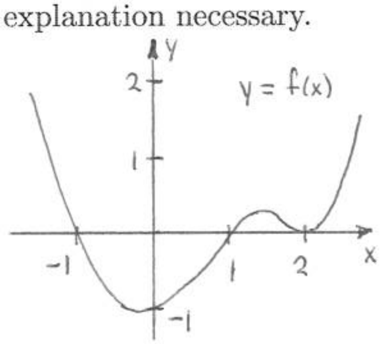
(3 pts) 14. A farmer will use 300 feet of fencing to form two rectangular pens against an existing wall, as pictured. Using the relation $3x + \ell = 300$, he finds that the pens will have total area $A = x(300 - 3x)$. He then completes the square to obtain the formula $A = -3(x - 50)^2 + 7500$. Using that formula, determine the maximum possible area and the dimensions that give that area. Include units in your answer.



Maximum possible area:

Dimensions x and ℓ that give that area:

(3 pts) 15. For the polynomial $f(x)$ whose graph is given, answer true or false. No explanation necessary.

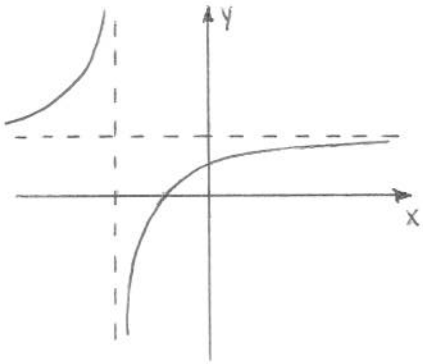


____ The degree of f is three.

____ The coefficient of the highest-degree term is positive.

____ $x + 1$, $x - 1$, and $x - 2$ are all factors of $f(x)$.

(4 pts) 16. The graph $y = (2x + 3)/(x + 3)$ is sketched below. Determine the asymptotes and intercepts, and show your work. Note: An asymptote is a line, note a number.



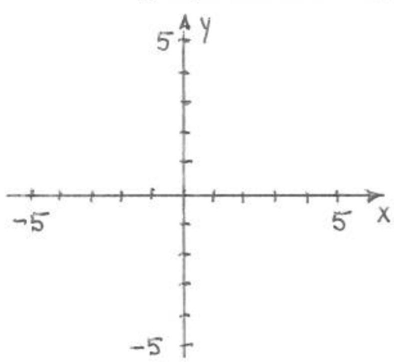
Vertical asymptote:

Horizontal asymptote:

x-intercept:

y-intercept:

(5 pts) 17. Graph $y = 2 - e^{-x}$ on your calculator, and copy the graph here. Then give the asymptote or asymptotes, and determine the intercepts (if there are any) in exact form. Note: An asymptote is a line, not a number



Asymptote(s):

Exact x-intercept(s), if any:

Exact y-intercept(s), if any:

(3 pts) 18. Find the domain of the function $f(x) = \log_2(4 - x^2)$. Show your work.

(3 pts) 19. Write $\ln\left(\dfrac{x\sqrt{x}}{x^2+1}\right)$ using sums and differences of simpler logarithmic expressions. Express the answer so that logarithms of products, quotients, and powers do not appear. Assume that $x > 0$.

(2 pts) 20. Given that $\log_b(2) = 0.43$, find $\log_b(\frac{1}{4})$.

(6 pts) 21. Solve the equation or inequality. Give your answers in exact form.

(a) $\log_2(x^2+1) = 3$

(b) $1 + 5e^x \leq 2$

(3 pts) 22. You deposit \$6000 in an account that pays 5% per annum compounded continuously. To the nearest tenth of a year, when will the balance reach \$20,000? Show your work.

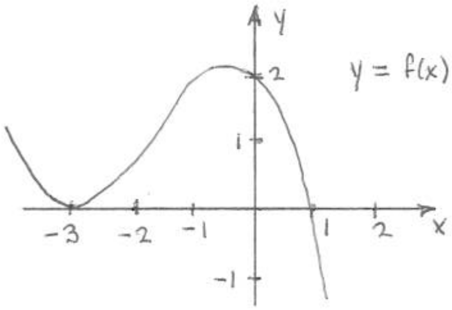
(4 pts) 23. Boise's population was 185,767 in 2000 and 205,671 in 2010. Assuming an exponential growth law $\mathcal{N} = \mathcal{N}_0 e^{kt}$, what will the population be in 2015? Show your work.

(4 pts) 24. Write in the form $a + bi$:

(a) $(3 + 2i)(4 - i)$

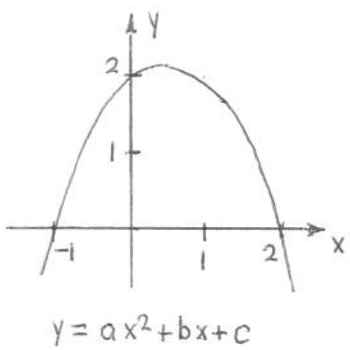
(b) $\dfrac{3 + 2i}{4 - i}$

(3 pts) 25. The graph of a cubic polynomial $f(x)$ is given. Use the graph to determine the roots of $f(x) = 0$ and their multiplicities.



Root	Multiplicity

(3 pts) 26. Find the quadratic function $y = ax^2 + bx + c$ whose graph is given.



(3 pts) 27. Express the polynomial $2x^2 - 3x + 1$ in the form $a(x - r_1) \cdots (x - r_n)$.

(6 pts) 28. Use the given root of the equation to find all the roots.

(a) $x^3 + 2x^2 - 3x - 10 = 0$; 2 is a root.

(b) $4x^4 + 15x^2 - 4 = 0$; $2i$ is a root.

(2 pts) 29. List the possible rational roots of $2x^3 + 5x - 3 = 0$. Then stop; do not test those numbers to see if they actually are roots.

(2 pts) 30. True or false. No explanation necessary.

____ (a) The fundamental theorem of algebra assures that there is at least one complex root of the equation $(2 + i)x^3 - ix + \sqrt{2} = 0$.

____ (b) Every polynomial equation with integer coefficients has at least one rational root.