

MATH 1143 EXAM ON CHAPTER 5
NOVEMBER 9, 2011

Instructions. Be sure to show your work! A correct solution with no supporting work might receive no credit. Give numerical answers in exact form, such as $5/7$ or $3\sqrt{5}$, unless the problem asks for a decimal approximation. There are 35 points altogether.

(3 pts) 1. Give the domain of the function. Interval notation is optional; rather than $(-\infty, 0) \cup [1, 5)$, for example, you may say “ $x < 0$ or $1 \leq x < 5$.”

(a) $y = \frac{1}{3^{x-2}}$

(b) $y = \log_3(x^2 - 4)$

(c) $y = (\log_3 x)^2 - 4$

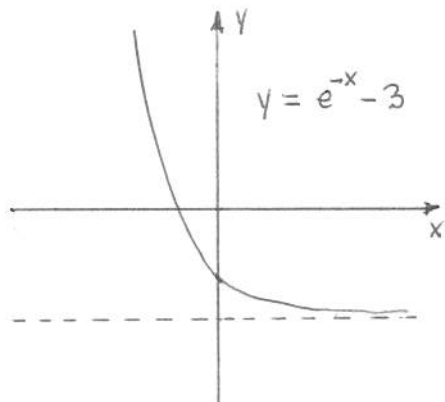
(2 pts) 2. Use properties of exponents to simplify the expression. Assume that $b > 0$.

(a) $(b^{\sqrt{2}})(b^{-\sqrt{2}})$

(b) $\frac{(b^{3/2})^4}{b}$

(2 pts) 3. Determine the intercepts of the graph $y = e^{-x} - 3$ in exact form.

x -intercept:



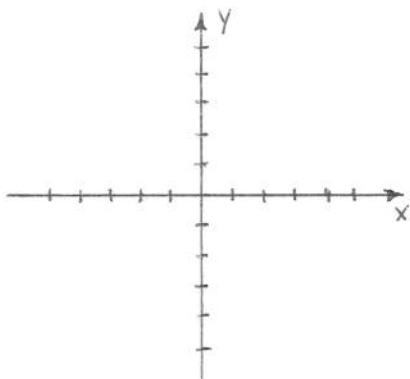
y -intercept:

(2 pts) 4. Write the equation in the specified form. Then stop! Do not solve the equation.

(a) Write the equation $e^{4t} = 5.7$ in logarithmic form.

(b) Write the equation $\log_5(t^2 + 3) = -2$ in exponential form.

(3 pts) 5. Sketch the graph $y = \ln(x + 1) + 2$. Then give the intercepts *in exact form* and the asymptote. Note: An asymptote is a line.



x -intercept:

y -intercept:

Asymptote:

(2 pts) 6. Circle the true statements.

$$e^2 < 10$$

$$2^{-\sqrt{5}} < 0$$

$$\log_3 \pi > 1$$

The function $f(x) = e^x$ is one-to-one.

(2 pts) 7. Write $2 \log_{10} 3 + \log_{10} x - \log_{10}(x + 2)$ as a single logarithm.

(2 pts) 8. Write $\ln \left(\frac{x(x+1)^2}{\sqrt{2x-1}} \right)$ using sums and differences of simpler logarithmic expressions in which logarithms of products, quotients, and powers do not appear.

(4 pts) 9. Solve the equation. Give your solutions in exact form.

(a) $\log_2(x + 1) = 3 + \log_2 x$

(b) $3^x = 5 \cdot 2^x$

(3 pts) 10. (a) What is the domain of the function $y = \log_{10}(x + 2)$?

(a) Solve the inequality $\log_{10}(x + 2) < 1$.

(2 pts) 11. You deposit \$2000 in an account with a 5% annual interest rate. To the cent, what will the balance be after 10 years if interest is compounded:

(a) quarterly?

(b) continuously?

(3 pts) 12. A principal of \$4000 is invested at 6% interest per annum, compounded continuously.

(a) What will the balance be after one year?

(b) What is the effective interest rate to the nearest hundredth of a percent? Suggestion: Use your answer to (a).

(c) In how many years (from the beginning) will the balance reach \$10,000? Round your answer to the nearest tenth of a year.

(2 pts) 13. The radioactive isotope Strontium-90 has a half-life of 28 years. If you get a 10-gram sample of it (as a present, say), how much will remain after 10 years? Round your answer to the nearest tenth of a gram.

(3 pts) 14. Use the exponential model $N = N_0e^{kt}$ for the following: In 2000, the United States had a population of 275.6 million and relative growth rate of 0.6% per year, and Pakistan had a population of 150.6 million and relative growth rate of 2.8% per year. If those growth rates remain the same:

(a) What will the populations of the United States and Pakistan be in 2020?

(b) In what year will the population of Pakistan first exceed that of the United States?