

**Final Exam****Math 143****Monday, May 7, 2007**

Name \_\_\_\_\_

Instructor \_\_\_\_\_ Section number \_\_\_\_\_

**Directions:** Show all work and simplify all answers. If you use your calculator to arrive at a conclusion, state what you did on your calculator. Give exact answers unless asked for a decimal approximation. Box your final answers. There are 200 points possible: budget your time on each problem effectively.

NPC means “no partial credit”.

Page	Points possible	Points earned
2	28	
3	32	
4	28	
5	21	
6	22	
7	20	
8	26	
9	23	
TOTAL	200	

1. [7 × 4 = 28 points]

(a) [4 points, NPC] Let  $H(t) = 2t$ . Compute  $H(t + 1)$ . Simplify your answer.

\_\_\_\_\_

(b) [4 points, NPC] Fill in the blank with one of the options (i), (ii), (iii) or (iv).  
The function  $f(x) = 3 - x^2$  is a \_\_\_\_\_ function.

- i. linear
- ii. quadratic
- iii. logarithmic
- iv. exponential

\_\_\_\_\_

(c) [4 points, NPC] Fill in the blank with one of the options (i), (ii), or (iii). A  
polynomial of degree five \_\_\_\_\_ has three turning points.

- i. always
- ii. sometimes
- iii. never

\_\_\_\_\_

(d) [4 points, NPC] Fill in the blank with one of the options (i), (ii), or (iii). A  
polynomial  $p(x)$  of degree five \_\_\_\_\_ has a vertical asymptote.

- i. always
- ii. sometimes
- iii. never

\_\_\_\_\_

(e) [4 points, NPC] Simplify  $e^{\frac{1}{2} \ln x}$ .

\_\_\_\_\_

(f) [4 points, NPC] What is the inverse function  $f^{-1}(x)$  to  $f(x) = \log_3 x$ ?

\_\_\_\_\_

(g) [4 points, NPC] Find the range of the function  $f(x) = 2^x$ .

\_\_\_\_\_

2. [8 × 4 = 32 points]

(a) [4 points, NPC] Simplify  $\log_2 \frac{1}{8}$ .

\_\_\_\_\_

(b) [4 points, NPC] True or false:  $\ln(x + y) = \ln x + \ln y$  for all  $x, y > 0$ .

\_\_\_\_\_

(c) [4 points, NPC] Fill in the blank with one of the options (i), (ii), or (iii). A one-to-one function  $f(x)$  \_\_\_\_\_ has an inverse function  $f^{-1}(x)$ .

- i. always
- ii. sometimes
- iii. never

\_\_\_\_\_

(d) [4 points, NPC] Give the exact equation for the vertical asymptote to the graph  $y = \frac{150}{7x + 3}$ .

\_\_\_\_\_

(e) [4 points, NPC] Give the exact equation for the horizontal asymptote to the graph  $y = \frac{150}{7x + 3}$ .

\_\_\_\_\_

(f) [4 points, NPC] True or false:  $(x - 1)$  is a factor of the polynomial  $p(x) = x^{99} - 99$ .

\_\_\_\_\_

(g) [4 points, NPC] If  $z = 4 + 2i$ , find  $\bar{z}$ .

\_\_\_\_\_

(h) [4 points, NPC] Write  $(1 - 3i)^2$  in the form  $a + bi$ .

\_\_\_\_\_

3. [8 points] Let  $f(x) = 3x - 2$  and let  $g(x) = x^2$ .

(a) [4 points] Find  $(f \circ g)(x)$ .

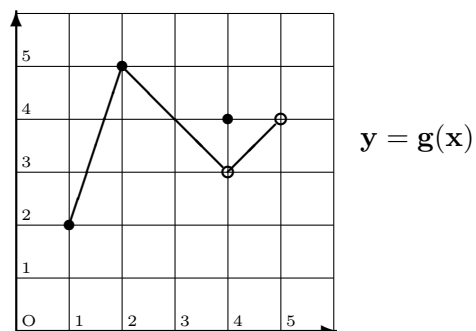
(b) [4 points] Find  $(f + g)(-1)$ .

4. [10 points] Find the inverse function  $f^{-1}(x)$  of  $f(x) = e^{2x} - 1$ .

$$f^{-1}(x) = \underline{\hspace{10em}}$$

5. [10 points] Let  $f(x) = 3x - 2$ . Find  $\frac{f(x+h) - f(x)}{h}$ . Simplify your answer.

6. [11 points] The graph of  $y = g(x)$  is given.



(a) [4 points] Find the domain of  $g$ .

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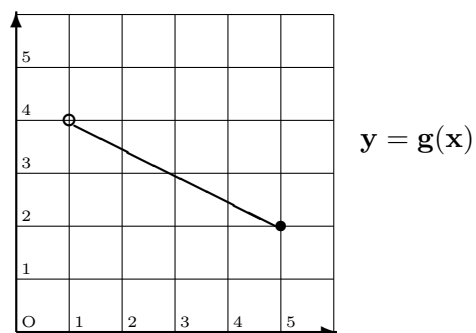
(b) [4 points] Find the range of  $g$ .

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(c) [3 points] Find  $g(4)$ .

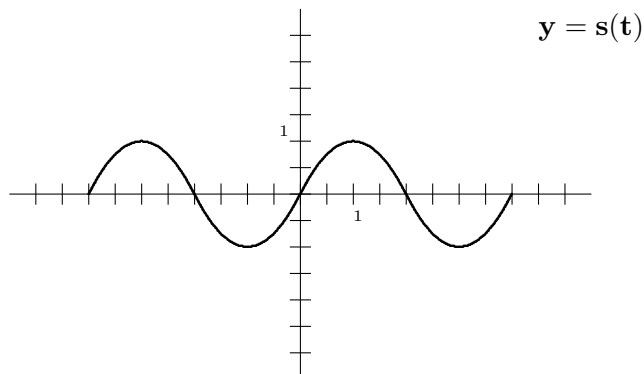
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7. [10 points] The graph of  $y = g(x)$  is given. Plot the inverse function on the same axes.

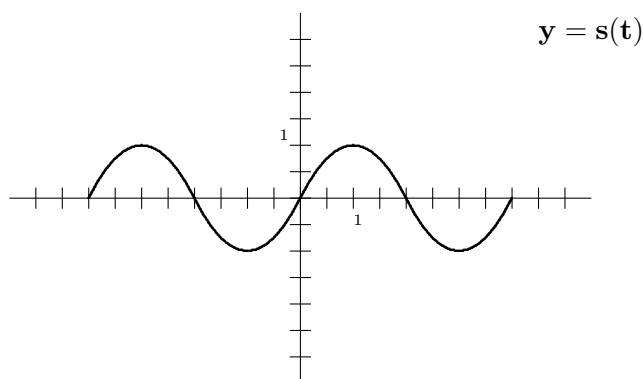


8. [12 points]

(a) The graph of  $y = s(t)$  is given. Graph  $y = 3s(t - 1)$  on the same axes.



(b) The graph of  $y = s(t)$  is given. Graph  $y = -s(t) + 1$  on the same axes.



9. [10 points] The owner of a small business buys a new truck for \$10,000. After eight years, the vehicle has resale value of \$2,000. Assuming linear depreciation, find a formula  $V(t)$  for the value of the vehicle after  $t$  years, where  $0 \leq t \leq 8$ .

$V(t) =$  \_\_\_\_\_

10. [10 points] Express the circumference  $C$  of a circle as a function of its area  $A$ .

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11. [10 points] A baseball is thrown straight up into the air. Its height  $h$  as a function of time is given by  $h = -16t^2 + 32t$ , where  $h$  is in feet and  $t$  is in seconds. Find the maximal height of the ball and the time at which that height is attained. Explain your answer, justifying all steps. Include units in your answer.

maximal height: \_\_\_\_\_

time attained: \_\_\_\_\_

12. [13 points] Write the quantity below using sums and differences of simpler logarithmic expressions. Express the answer so that logarithms of products, quotients, and powers do not appear. Simplify your answer.

$$\ln \left( \frac{x}{(3x+1)^3 \sqrt{x+2}} \right)$$

13. [13 points] Solve the inequality  $(1/2)^{x-3} < 8$ . Justify your answer, showing all steps. Express your final answer in interval notation.

14. [13 points] An investment fund has an interest rate of 5% per annum, compounded monthly. What principal  $P$  will grow to \$20,000 in 10 years under these conditions? Round your answer to the nearest cent.

$P$  is \_\_\_\_\_

15. [10 points] Express the polynomial  $x^2 + 2x + 2$  in the form  $a(x - r_1)(x - r_2)$ . Express complex numbers in standard form  $a + bi$ .

$$a(x - r_1)(x - r_2) = \underline{\hspace{10cm}}$$