



5. [15 points] A cube has side length  $x$ .

(a) [3 points] Express the surface area  $S$  of the cube as a function of  $x$ .

(b) [6 points] Express the volume  $V$  of the cube as a function of the cube's surface area  $S$ .

(c) [6 points] Express the surface area  $S$  of the cube as a function of its volume  $V$ .

6. [15 points] The difference of two numbers is 1. What is the smallest possible value of the sum of their squares?

7. [20 points] Let  $g(x) = (x - 1)(x + 1)(x - 3)^2$ .

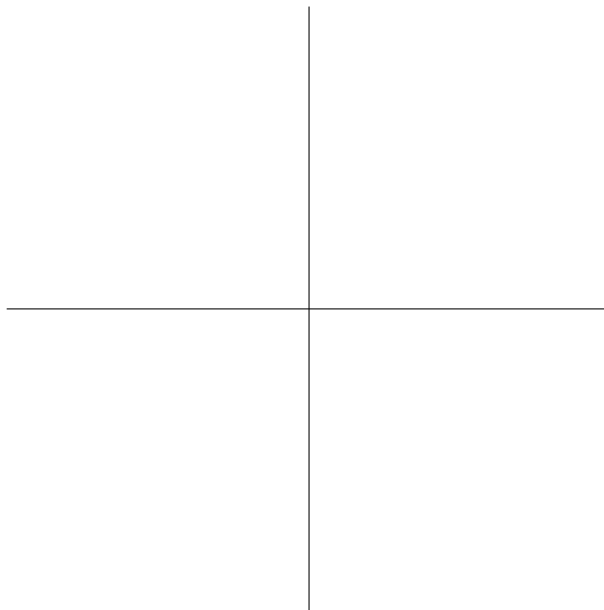
(a) Find the  $x$ -intercepts.

(b) Find the  $y$ -intercept.

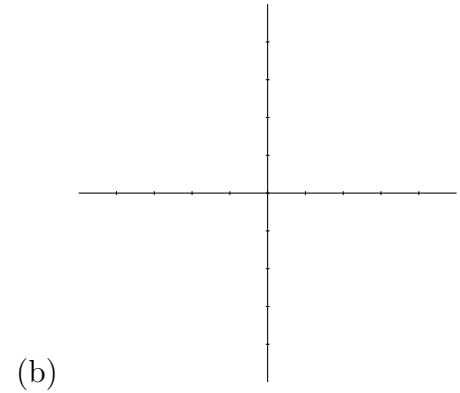
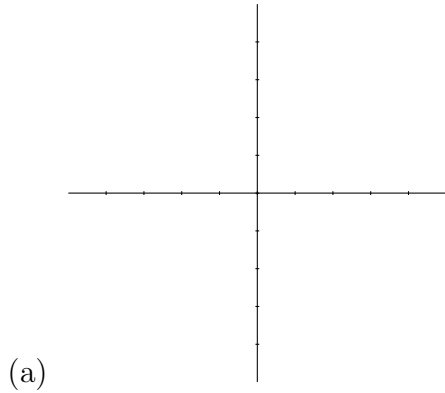
(c) Do a sign analysis. Either show your table, or indicate which test points you used on your number line.

(d) Describe the behavior of the function at each  $x$ -intercept. Specify your results with a sketch.

(e) Graph  $g$ . Shade in the excluded regions on your graph. Add tick marks on the axes to indicate scale.



8. [8 points] In parts (a) and (b), give a reason why the graph can not represent a polynomial function with leading term  $-2x^3$ . State the mathematical principle behind your answer.



9. [12 points] Find the inverse function of  $f(x) = \frac{4}{x^3 + 1}$ . [If you have time, check your answer by showing  $(f \circ f^{-1})(x) = x$ ]