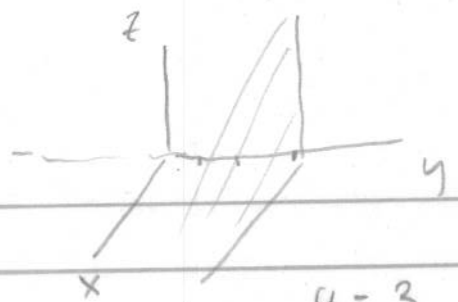
Math 275
Solutions MT 1

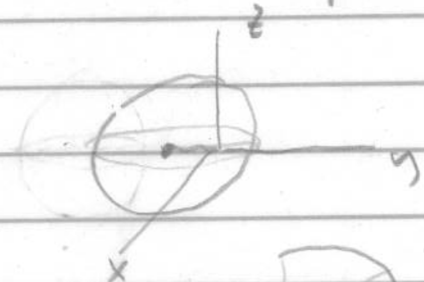
(a) (6)



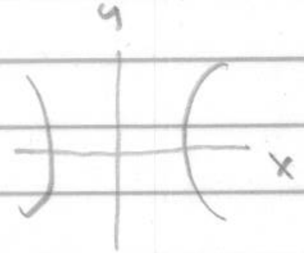
$y = 3$ describes a plane parallel to the x-z plane.

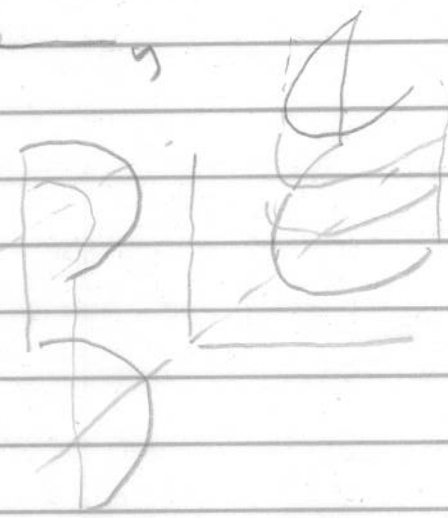
(b) (7) $x^2 + y^2 + 6y + z^2 = x^2 + (y+3)^2 - 9 + z^2 = 9$

$x^2 + (y+3)^2 + z^2 = 18$ sphere radius $3\sqrt{2}$ center $(0,-3,0)$



(c) (7)





$x^2 + y^2 = 1$

cylinder over a hyperbola.

(2) (a) (5) $\langle 1,2,3\rangle \cdot \langle -1,4,2\rangle = -1 + 8 + 6 = 13$

(5) $\vec{v} \times \vec{w} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ -1 & 4 & 2 \end{vmatrix} = 2\vec{i} - 3\vec{j} + 4\vec{k}$
$-12i - 2j + 2\vec{k}$

$= \langle -8, -5, 6\rangle$

(5) (c) $|\vec{v}| = \sqrt{1+4+9} = \sqrt{14}$ $\quad |\vec{w}| = \sqrt{1+16+4} = \sqrt{21}$

$$\therefore \cos\theta = \vec{v}\cdot\vec{w} / |\vec{v}|\cdot|\vec{w}| = \frac{13}{\sqrt{14}\sqrt{21}} = \frac{13}{7\sqrt{6}}$$

$$\theta = \arccos\left(\frac{13}{7\sqrt{6}}\right)$$

(5) (d) $\vec{v}\cdot\vec{v}\times\vec{w} = 0 = \vec{w}\cdot(\vec{v}\times\vec{w})$

so $\vec{v}/|\vec{v}| = \frac{\langle 1,2,3\rangle}{\sqrt{14}}$, $\vec{w}/|\vec{w}| = \frac{\langle -1,4,2\rangle}{\sqrt{21}}$

are unit vector perpendicular to $\vec{v}\times\vec{w}$

(3) (a) $\vec{r} = \vec{r}_0 + t\vec{v}$ becomes. $\vec{r} = \langle 1,-2,-6\rangle + t\langle 5,-3,2\rangle$ (5)

Symmetric equ'n is

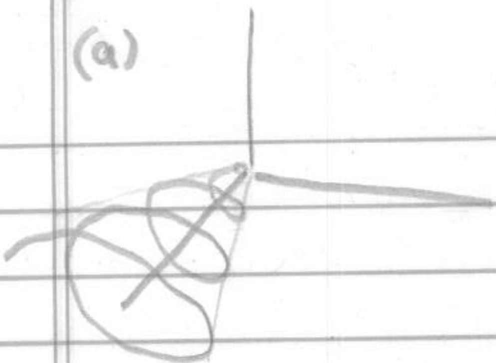
$$\frac{x-1}{5} = -\frac{(y+2)}{3} = \frac{z+6}{2}. \quad (5$$

(b) $\langle 2,-4,3\rangle\cdot\left(\vec{r} - \langle 1,0,1\rangle\right) = 0.$ (5)

$2(x-1) - 4y + 3(z-1) = 0$ scalar equation

4)

(a)



(b) $x^2 - y^2 - z^2$

$= t^2 - (t^2\cos^2 t + t^2 \sin^2 t)$

$= t^2 - t^2 = 0$

The surface is a cone.

$\vec{r}\,' = \langle 1, \cos t, \sin t \rangle + \langle 0, -t\sin t, t\cos t \rangle$

$|\vec{r}\,'| = \sqrt{2 + t^2}$

$\dfrac{\vec{r}\,'}{|\vec{r}\,'|} = \dfrac{1}{\sqrt{2+t^2}} \langle 1, \cos t - t\sin t, \sin t + t\cos t \rangle$

$\vec{r}\,''(t) = 2\langle 0, -\sin t, \cos t \rangle - t\langle 0, \cos t, \sin t \rangle$

(5) (a) $f_x = y\sin(xz) + xyz\cos(xz)$

$f_y = x\sin(xz)$, $f_z = x^2 y\cos(xz)$

(b) $f_{xx} = zy\cos(xz) + yz\cos(xz) - xyz^2\sin(xz)$

$= 2yz\cos(xz) - xyz^2\sin(xz)$

$f_{xz} = xy\cos(xz) + xy\cos(xz)$

$- x^2yz\sin(xz)$

$= 2xy\cos(xz) - x^2yz\sin(xz)$