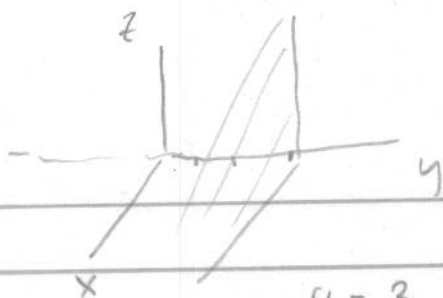


(a) (b)

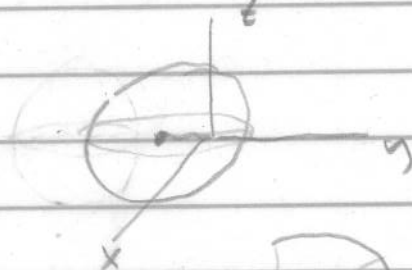


$y=3$  describes a plane parallel to the  $x-z$  plane.

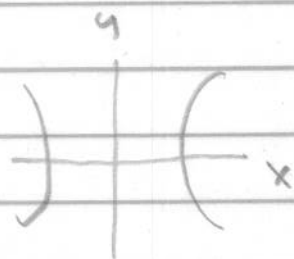
(b) (f)

$$x^2 + y^2 + 6y + z^2 = x + (y+3) - 9 + z^2 = 9$$

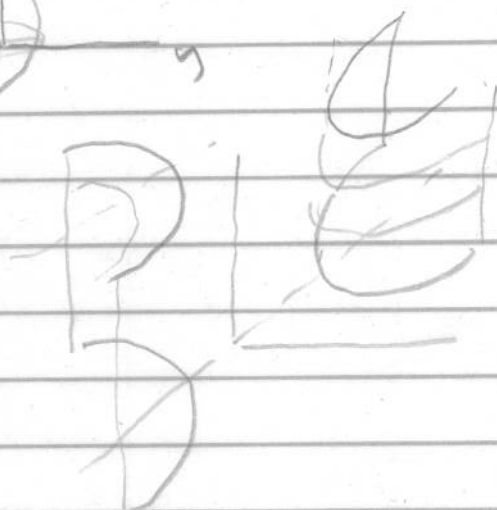
$$x^2 + (y+3)^2 + z^2 = 18 \quad \text{Sphere radius } 3\sqrt{2} \text{ center } (0, -3, 0)$$



(c) (f)



$$x^2 - y^2 = 1$$

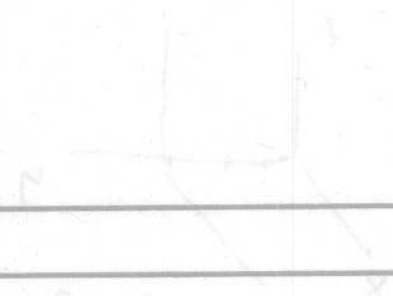


cylinder over a hyperbola.

(2) (a) (5)  $\langle 1, 2, 3 \rangle \cdot \langle -1, 4, 2 \rangle = -1 + 8 + 6 = 13$

(5)  $\vec{u} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ -1 & 4 & 2 \end{vmatrix} = 2\vec{i} - 3\vec{j} + 4\vec{k} - 12\vec{i} - 2\vec{j} + 2\vec{k} = \langle -8, -5, 6 \rangle$

MATH 242  
Solutions MT 1



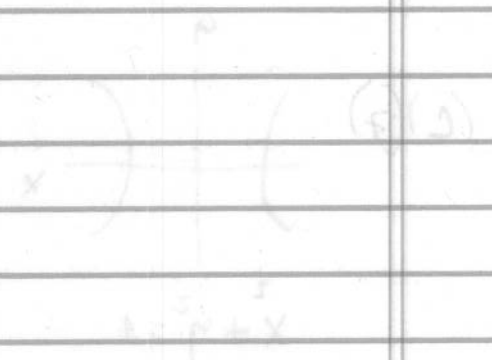
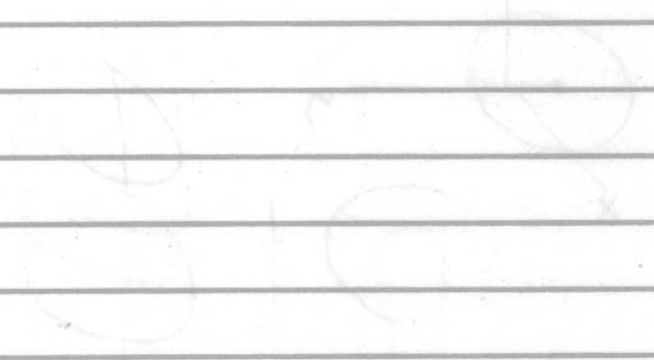
(a) (b)

Find a plane parallel to the  $xz$ -plane

(b) (i)  $P = 5 + P - (8 + P) + X = 5 + P + X - 8 - P = X - 3$

$X + (P + 3) + 5 = 18$  plane through  $(8, 5, 5)$  center

$(0, 3, 0)$



Find a plane parallel to the  $xz$ -plane

(b) (ii)  $(1, 2, 3) \cdot (-1, 1, 1) = -1 + 2 + 3 = 4$

(c)  $\vec{v} \times \vec{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ -1 & 1 & 1 \end{vmatrix} = \mathbf{i}(2-3) - \mathbf{j}(1-3) + \mathbf{k}(1-2) = -\mathbf{i} + 2\mathbf{j} - \mathbf{k}$

$(-1, 2, -1) \cdot (0, 3, 0) = -3$

$$(2) (a) \quad |\vec{v}| = \sqrt{1+4+9} = \sqrt{14} \quad |\vec{w}| = \sqrt{1+16+4} = \sqrt{21}$$

$$\therefore \cos \theta = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| \cdot |\vec{w}|} = \frac{13}{\sqrt{14} \sqrt{21}} = \frac{13}{7\sqrt{6}}$$

$$\theta = \arccos \left( \frac{13}{7\sqrt{6}} \right)$$

$$(3) (d) \quad \vec{v} \cdot (\vec{v} \times \vec{w}) = 0 = \vec{w} \cdot (\vec{v} \times \vec{w})$$

$$\text{So } \frac{\vec{v}}{|\vec{v}|} = \frac{\langle 1, 2, 3 \rangle}{\sqrt{14}}, \quad \frac{\vec{w}}{|\vec{w}|} = \frac{\langle -1, 4, 2 \rangle}{\sqrt{21}}$$

are unit vector perpendicular to  $\vec{v} \times \vec{w}$

$$(3) (a) \quad \vec{r} = \vec{r}_0 + t\vec{v} \text{ becomes } \vec{r} = \langle 1, -2, -6 \rangle + t\langle 5, -3, 2 \rangle$$

Symmetric eqn's is (5)

$$\frac{x-1}{5} = -\frac{(y+2)}{3} = \frac{z+6}{2} \quad (5)$$

$$(b) \quad \langle 2, -4, 3 \rangle \cdot (\vec{r} - \langle 1, 0, 1 \rangle) = 0 \quad (5)$$

$$2(x-1) - 4y + 3(z-1) = 0 \quad \text{scalar equation}$$

$$|\vec{r}| = \sqrt{1+1+1} = \sqrt{3} \quad |\vec{u}| = \sqrt{1+1+1} = \sqrt{3} \quad (a) \quad \vec{r} = \vec{u}$$

$$\cos \theta = \frac{\vec{r} \cdot \vec{u}}{|\vec{r}| |\vec{u}|} = \frac{3}{\sqrt{3} \sqrt{3}} = 1 \quad \therefore \theta = 0^\circ$$

$$\vec{v} \cdot \vec{w} = 0 = \vec{v} \times \vec{w} \quad (b) \quad \vec{v} = \vec{w}$$

$$\cos \theta = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| |\vec{w}|} = \frac{3}{\sqrt{3} \sqrt{3}} = 1 \quad \therefore \theta = 0^\circ$$

we want vector perpendicular to  $\vec{v} \times \vec{w}$

$$\vec{r} = \vec{i} + \vec{j} + \vec{k} \quad \text{because } \vec{r} = (1, 1, 1) = \vec{i} + \vec{j} + \vec{k} \quad (c) \quad \vec{r} = \vec{i} + \vec{j} + \vec{k}$$

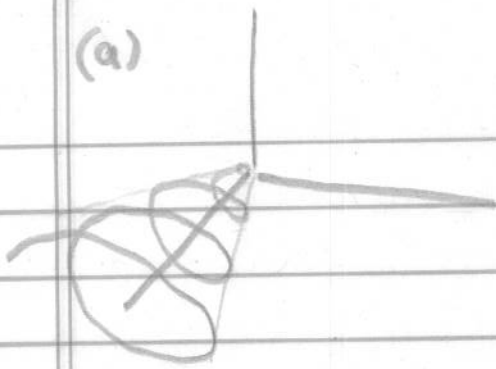
$$\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{3}$$

$$\langle \vec{r}, -\vec{i} + 2\vec{j} + 3\vec{k} \rangle = 0 \quad (d) \quad \vec{r} = \vec{i} + \vec{j} + \vec{k}$$

$$r(x-1) - r(y+2) + r(z-3) = 0$$

4)

(a)



$$\begin{aligned}
 (b) \quad x^2 - y^2 - z^2 &= t^2 - (t^2 \cos^2 t + t^2 \sin^2 t) \\
 &= t^2 - t^2 = 0
 \end{aligned}$$

The surface is a cone.

$$\vec{r}' = \langle 1, \cos t, \sin t \rangle + \langle 0, -t \sin t, t \cos t \rangle$$

$$|\vec{r}'| = \sqrt{2+t^2}$$

$$\frac{\vec{r}'}{|\vec{r}'|} = \frac{1}{\sqrt{2+t^2}} \langle 1, \cos t - t \sin t, \sin t + t \cos t \rangle$$

$$\vec{r}''(t) = 2 \langle 0, -\sin t, \cos t \rangle - t \langle 0, \cos t, \sin t \rangle$$

$$(5) (a) \quad f_x = y \sin(xz) + xy z \cos(xz)$$

$$f_y = x \sin(xz), \quad f_z = x^2 y \cos(xz)$$

$$\begin{aligned}
 (b) \quad f_{xx} &= zy \cos(xz) + yz \cos(xz) - xy z^2 \sin(xz) \\
 &= 2yz \cos(xz) - xy z^2 \sin(xz)
 \end{aligned}$$

$$f_{xz} = xy \cos(xz) + xy \cos(xz)$$

$$- x^2 y z \sin(xz)$$

$$= 2xy \cos(xz) - x^2 y z \sin(xz)$$

$$\begin{aligned}
 & \frac{d}{dt} (x^2 + y^2) = 2x \dot{x} + 2y \dot{y} \\
 & = 2x(-y) + 2y(x) \\
 & = -2xy + 2xy = 0
 \end{aligned}$$

The surface is a cone

$$\vec{r} = \langle t \cos t, t \sin t, t \rangle + \langle 0, -t \sin t, t \cos t \rangle$$

$$|\vec{r}'| = \sqrt{2 + t^2}$$

$$\frac{1}{|\vec{r}'|} \langle t \cos t - t \sin t, t \sin t + t \cos t, 1 \rangle$$

$$\vec{r}''(t) = \langle 0, -\sin t, \cos t \rangle - \langle 0, \cos t, \sin t \rangle$$

$$\vec{r}'(x) = x \sin(x) + x^2 \cos(x)$$

$$f(x) = x \sin(x) + x^2 \cos(x)$$

$$f'(x) = \sin(x) + x \cos(x) + 2x \cos(x) - x^2 \sin(x)$$

$$= 3x \cos(x) - x^2 \sin(x)$$

$$f''(x) = 3 \cos(x) - 2x \sin(x) - 2x \sin(x) - x^2 \cos(x)$$

$$= 3 \cos(x) - 4x \sin(x) - x^2 \cos(x)$$

$$= 3 \cos(x) - x^2 \sin(x) - x^2 \cos(x)$$