Midterm 1, Math 275

1. Draw a rough sketch of the set of points in three dimensional space which satisfy the following equations:

   (a) $y = x$

   (b) $(x^2/9) + (z^2/4) = 1$

   (c) $x^2 + y^2 - 8x = -12$

   (d) $x^2 + y^2 - z^2 = -1$.

2. Let $\vec{v} = \langle 1, -1, 7 \rangle$, $\vec{w} = \langle 3, 4, 1 \rangle$. Find the following:

   (a) $\vec{v} \cdot \vec{w}$

   (b) $\vec{v} \times \vec{w}$

   (c) The angle between $\vec{v}$ and $\vec{w}$.

   (d) A unit vector which is perpendicular to both $\vec{v}$ and $\vec{w}$.

3. (a) Find the vector equation and the symmetric equation of the line through the point $\langle 2, 1, 2 \rangle$ which is in the direction of the vector $\langle 3, -4, 5 \rangle$.

   (b) Find the vector equation and the scalar equation of the plane containing $\langle 2, 1, 8 \rangle$ which is orthogonal to the vector $\langle 4, 3, 2 \rangle$.

4. Consider the curve given by the vector valued function

$$\vec{r}(t) = \langle t, t^2, t^4 \rangle.$$

   (a) Draw a rough sketch of the curve.

   (b) Show that the curve lies on the surface $z = y^2$. (What does this surface look like?).

   (c) Compute a *unit* tangent vector to the curve.

   (d) Find that second derivative (acceleration).

5. Compute the partial detivatives f_x, f_y, f_z for the following functions $f(x, y, z)$. Also compute the gradients of these functions.

   (a) $f(x, y, z) = (2x + 3y + 4z)^{10}$ (b) $f(x, y, z) = xy/(xz + z^2)$
   (c) $f(x, y, z) = \sin(xyz)$

6. For the function $f(x, y) = x^2y^2 - xe^y$, compute the partial derivatives f_{xx}, f_{xy} and f_{yy}.

7. Show that the function $u(x, y) = e^x \sin y$ is a solution of the partial differential equation $u_{xx} + u_{yy} = 0$.