

MATH 143 — SLANT ASYMPTOTES

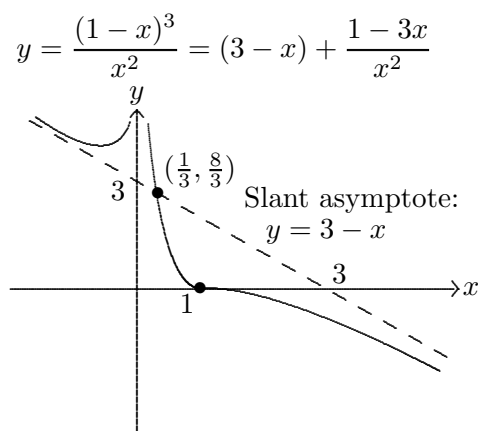
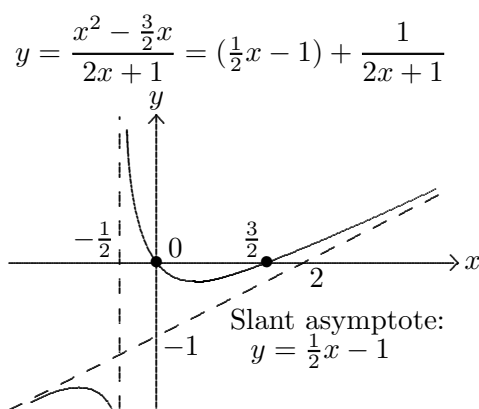
We have seen that a rational function $f(x) = \frac{p(x)}{d(x)}$ will have a horizontal asymptote if the degree of the numerator p is less than or equal to the degree of the denominator d . In particular, if the degree of p is strictly less than that of d , then the x -axis will be the horizontal asymptote—a geometrical condition that can be expressed analytically by saying $f(x) \rightarrow 0$ as $x \rightarrow \infty$ and as $x \rightarrow -\infty$.

If the degree of p is greater than or equal to the degree of d , then long division can be used to obtain more accurate information about the large scale behavior of the rational function. Recall that $p(x)$ divided by $d(x)$ gives a quotient $q(x)$ and a remainder $r(x)$, provided that $p(x) = d(x) \cdot q(x) + r(x)$ and provided that the degree of r is strictly less than the degree of d . In terms of rational functions, we have

$$f(x) = \frac{p(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}.$$

Because of the degree condition on r , it is clear that $\frac{r(x)}{d(x)}$ approaches zero as $x \rightarrow \pm\infty$, so that $f(x)$ and $q(x)$ are close to each other when $|x|$ is large. Thus the graph of the rational function $f(x)$ is **asymptotic** to the graph of the polynomial $q(x)$ as $x \rightarrow \pm\infty$. In other words, the two graphs are close to each other as $x \rightarrow \infty$ and as $x \rightarrow -\infty$. In the special case where the degree of p is one more than the degree of d , the quotient is a linear function, whose graph is a nonhorizontal line in the plane. That line is called an **oblique** or **slant** asymptote to the graph of the particular rational function.

Examples:



Problems: Determine all intercepts and asymptotes for the graphs of the following rational functions and use that information to help you sketch the graphs of the functions.

(a) $f(x) = \frac{2x^2}{1-x}$

(b) $f(x) = \frac{x^3 - 3x^2}{x^2 - 1}$

(c) $f(x) = \frac{(2+x)(2-3x)}{(2x+3)^2}$

(d) $f(x) = \frac{x^3 - 1}{x^2 - x - 2}$

(e) $f(x) = \frac{x^2 - 2x}{x^3 + 1}$

(f) $f(x) = \frac{1 - x^3}{x}$

(g) $f(x) = \frac{x^3 - 1}{2(x^2 - 1)}$

(h) $f(x) = \frac{x^4 - 2x^3 + 1}{x^2}$

ANSWERS TO SLANT ASYMPTOTE PROBLEMS

