5.1 Exponential functions—the base of an exponential function, domain, range, graph, asymptote, large scale behavior, rules for exponents, simplifying expressions.

5.2 The number e and the special role of the exponential to the base e.

5.3 Logarithms to the base b and the natural logarithm—definition, domain, range, asymptote, intercept, graphs, inverse relation.

5.4 The basic properties of the logarithm function—the five basic properties and the change of base formulas must be learned.

5.5 Solving equations and inequalities involving logarithm and exponential functions, extraneous solutions.

5.6 Compound interest, continuous compounding, the doubling time.

5.7 Exponential growth and decay, half life, doubling time.

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1. Multiple choice: The domain of $f(x) = \ln(2-7x)$ is

   a. $(\frac{2}{7}, \infty)$ b. $(-\frac{2}{7}, \infty)$ c. $(-\infty, 0)$ d. $(-\infty, \ln(2)/7)$ e. something else

2. Multiple choice: If $f(x) = \ln(2x) - 1$ then the inverse function $f^{-1}(x)$ equals

   a. $(e^x + 1)/2$ b. $\frac{1}{2}e^{x+1}$ c. $-\frac{1}{2}e^x$ d. $\frac{1}{2}\ln(x+1)$ e. $1/(\ln(2x) - 1)$

3. Sketch graphs of the following functions. Be sure to determine all intercepts and asymptotes.

   a. $y = \ln(2-x)$ b. $y = 2 - e^x$ c. $y = 2^{1-x} - 1$ d. $y = \ln|x| - 1$

4. State the rules for logarithms and exponents.

5. Solve the following equations exactly:

   a. $\ln(3x+1) = 2 + \ln(x)$ b. $\log_2(2x+1) + \log_2(3x+1) = 1$ c. $2^{x-3} = e^{-2x}$

6. Solve the inequality $\log_2(3-2x) \le 1$.

7. Simplify $\ln(e^{\ln 2}) + \ln\sqrt{e} - \ln(e^2\sqrt{2})$.

8. Determine rational numbers a, b, c, and d so that

   a. $x^a = \dfrac{1}{x \cdot \sqrt{x}}$ b. $x^b = \dfrac{x \cdot (x^2)^3}{x^3 \cdot \sqrt{x^6}}$ c. $x^c = \dfrac{\sqrt{x^3} \cdot x^3}{x^2 \cdot \sqrt[3]{x^2}}$ d. $x^d = \dfrac{x \cdot (\sqrt{x})^3}{\sqrt[3]{x^4} \cdot \sqrt[6]{x^7}}$

9. Which of the following is/are valid identities? (assume $a, b > 0$)

   a. $(\ln a)^b = b \ln a$ b. $\log_2(a+b) = \log_2(a) \cdot \log_2(b)$ c. $(e^a)^b = e^{a+b}$ d. $a^{\ln b} = b^{\ln a}$

10. An account earns 5% interest, compounded monthly.

    a. If \$2,000 is deposited in the account, how much will be in the account after 6 years, assuming that there are no additional deposits or withdrawals.

    b. What initial deposit would be needed to have \$3,000 in the account after 6 years?

11. It is found that 3 grams of a 358 gram sample of a radioactive material have decayed after 5 years. What is the half-life of the material, computed to the nearest year?

12. A bacterial population grows from 4,000 to 5,000 in 4 hours. Assuming exponential growth, when will the population reach 10,000 ?

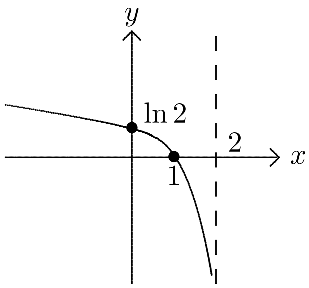
13. A recent experiment at the National Superconducting Cyclotron Laboratory in East Lansing, Michigan showed that the half life of titanium-44 is 59.2 years. How much of a $37\mu g$ sample of titanium-44 decayed during the six month experiment?

14. In 3 hours, the population of Tribles on the Enterprise has grown from 4 to 37. Estimate the population after an additional 7 hours, assuming:

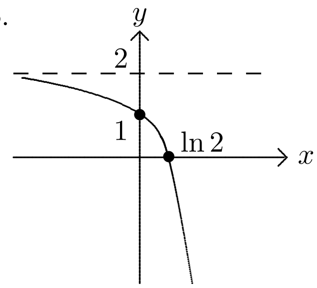
    a. linear growth b. exponential growth

ANSWERS

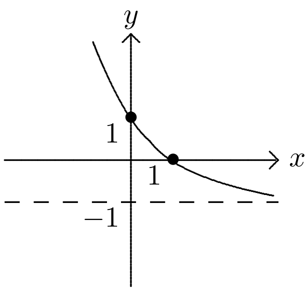
1. e. Something else: $(-\infty, \frac{2}{7})$
2. b. $\frac{1}{2}e^{x+1}$
3. a.



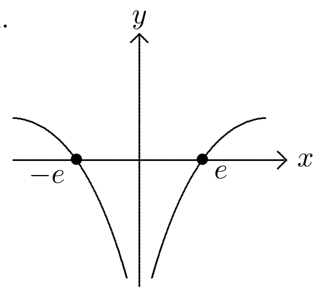
b.



c.



d.



4. **Exponents:** For all positive numbers a and b and all real numbers x and y

$$(i)\ \ a^x \cdot a^y = a^{x+y} \qquad (ii)\ \ a^x/a^y = a^{x-y} \qquad (iii)\ \ (a^x)^y = a^{x \cdot y}$$
$$(iv)\ \ (a \cdot b)^x = a^x \cdot b^x \qquad (v)\ \ (a/b)^x = a^x/b^x \qquad (vi)\ \ a^0 = 1$$

Logarithms: For all positive numbers a, b, x, and y and real numbers u,

$$(i)\ \ \log_b(xy) = \log_b(x) + \log_b(y) \qquad (ii)\ \ \log_b(x/y) = \log_b(x) - \log_b(y)$$
$$(iii)\ \ \log_b(x^u) = u \cdot \log_b(x) \qquad (iv)\ \ \log_b(1) = 0 \qquad (v)\ \ \log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

5. a. $x = \dfrac{1}{e^2 - 3}$ b. $x = \dfrac{1}{6}$. The “solution” $x = -1$ is extraneous. c. $x = \dfrac{3\ln(2)}{2 + \ln(2)}$
6. $[\frac{1}{2}, \frac{3}{2})$ (Note: domain)
7. $-\frac{3}{2} + \frac{1}{2}\ln(2)$
8. a. $-\frac{3}{2}$ b. 1 c. $\frac{11}{6}$ d. 0
9. Only part d is a valid identity, as can be verified by taking the natural logarithm of both sides. Parts a, b, and c are all erroneous versions of standard identities.
10. a. $2000\left(1 + \dfrac{0.05}{12}\right)^{12 \cdot 6} = \$2698.04$

    b. $P_0\left(1 + \dfrac{0.05}{12}\right)^{12 \cdot 6} = \$3000.00$ will give $P_0 = 2223.84$.
11. $t_{1/2} = \dfrac{5\ln(1/2)}{\ln(355/358)} = 412$ years
12. $t = \dfrac{4\ln(10000/4000)}{\ln(5000/4000)} = 16.425$ hours after the population was 4000.
13. $37\left(1 - e^{\frac{\ln(0.5)}{59.2} \times 0.5}\right) = .2160\ \mu g$
14. a. Linear $P(t) = 4 + 11t$ gives $P(10) = 114$.

    b. Exponential $P(t) = 4 \cdot (37/4)^{t/3}$ gives $P(10) = 6646$.