4.1 Linear functions—what they are and the connection between slopes and rates of change.
Economic examples: marginal cost, linear depreciation, average velocity, ...
Linear interpolation (i.e., two points) vs. a regression line. Using linear models.

4.2 The algebra and geometry of quadratic functions.
Parabolas, vertex and axis of symmetry, extreme values, intercepts. SKIP 4.3
Completing the square and the vertex form of a quadratic function.

4.4 Sections 4.4 and 4.5 involve a variety of word problems. Ideas to consider include the use of diagrams, choosing variables, expressing functions in terms of variables, and the use of constraints to express functions in terms of a single variable. Basic geometrical formulas that should be learned include: the pythagorean theorem and distance in the coordinate plane; areas and perimeters of squares, circles, rectangles, and triangles; and the volumes and surface areas of spheres, cylinders, and rectangular boxes. Composite figures are also considered.

4.5 The problems in section 4.5 combine ideas from 4.2 and 4.4. They are all word problems where the extreme values can be understood by determining the vertex of a quadratic.

4.6 Polynomial functions and the degree of a polynomial.
The leading coefficient and large scale behavior.
Zeros, intercepts, factored form, behavior near multiple roots, sign determination.
Continuity, smoothness, and the fact that the number of turning points is less than the degree.

4.7 Rational functions—definition, domain, vertical asymptotes.
Large scale behavior, horizontal and slant asymptotes.
Factored form, intercepts, sign determination, graphing.

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1. Which of the following functions are polynomials? For those that are, give the degree; for those that are not, indicate why not.

   a. $f(x) = \frac{\pi}{2} \cdot x^3 - 4\sqrt{5}$

   b. $f(x) = (2x^3 - 4x + 1)^2/(2x + 3)$

   c. $f(x) = (3x^2 + 2x - 1)^4\,(2x + \sqrt{5})^3$

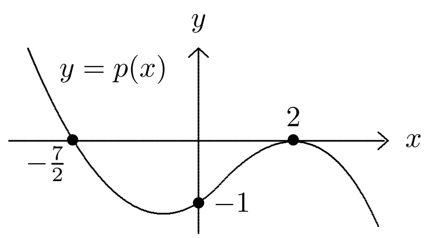
   d. $f(x) = 3x^2 + 2x^{-3} - 7x^4$

   e. $f(x) = (3x^3 - 4x + \pi)^2(2x + \sqrt{x})$

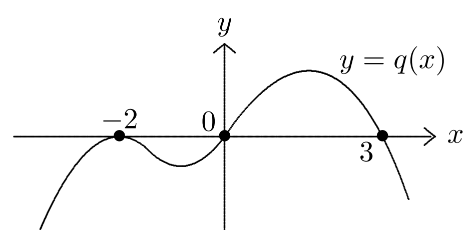
   f. $f(x) = -17$

2. Give a possible formula for a polynomial with the given graph.

   a.



   b.



3. For each of the following functions, describe the behavior of the function as $x \to \infty$ and as $x \to -\infty$ by one of the following: $f(x) \to +\infty$, $f(x) \to -\infty$, or $f(x) \to L$ where L is a particular real number whose value is expressed clearly.

   a. $f(x) = x^2 - x^3$

   b. $f(x) = \dfrac{2x^2 + 3}{x(4 - 3x)}$

   c. $f(x) = \dfrac{x^3}{2 - x}$

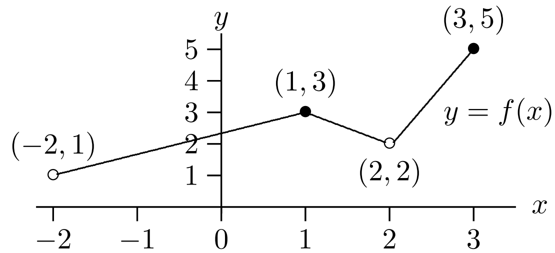
   d. $f(x) = \dfrac{(3x - 2)}{(2x + 1)^2}$

   e. $f(x) = \dfrac{(2 + 3x)(3 - 2x)^2}{x \cdot (1 - x)(x + 4)}$

   f. $f(x) = \dfrac{x \cdot (2 - 3x)^2}{(x + 4)(5 - 2x)}$.

4. Find the vertex of $y = \frac{1}{4}x^2 + 3x - 7$ by completing the square.

5. Give a piecewise definition for the function $f(x)$ with the following graph.



6. Sketch graphs of the following functions. Be sure to determine intercepts, asymptotes, sign, and large scale behavior.

   a. $f(x) = (x^2 + 2\,x)(x - 2)^2$

   b. $f(x) = \dfrac{x \cdot (2 - x)}{(x + 1)^2}$

   c. $f(x) = \dfrac{(x - 1)(3\,x + 4)}{(x + 1)(5 - 2\,x)}$.

   d. $f(x) = x + 1 - \dfrac{6}{x}$

7. Solve $\dfrac{3 - 2\,x}{x + 1} \leq 1$. Verify your result graphically.

8. Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(3, 0)$.

9. A box with a square bottom is to contain $1000\,in^3$. The material for the top and the sides of the box costs $5¢/in^2$ while the bottom costs $7¢/in^2$. There are also labor costs of \$2.00 per box. Assume that there are no other costs. Express the total cost of making a box as a function of the width of the base.

10. A farmer uses $1000\,ft$ of fence to create two separate pens, one in the shape of a square, the other in the shape of a rectangle that is twice as long in one direction as the other. Determine the total area of the two pens as a function of the length of a side of the square pen.

11. A ski area has fixed costs of \$12,000 per day and variable costs averaging \$4 per customer. They estimate that there will be $2000 - 25p$ customers per day when the ticket price is $\$p$.

    a. Estimate the area's daily costs as a function of the ticket price.

    b. Estimate the area's daily revenues as a function of p.

    c. What is the daily profit if the ticket price is \$30 ?

    d. What ticket price will maximize profits?

12. The production of solid waste in the United States has been increasing—from 82.3 million tons in 1960 to 139.1 million tons in 1980. Use a linear model to estimate the amount of waste produced in 1990 and 2000. What does the slope represent, and what are its units?

13. The table below shows the population of Idaho from 1950 to 1990. The equation of the regression line is $y = 11{,}130x - 21{,}145{,}000$, where x is the year and y is the population.

x (*year*)	1950	1960	1970	1980	1990
y (*population*)	588,637	667,191	713,015	944,127	1,006,749

    a. Use the regression line to estimate the population of Idaho in 2010.

    b. The 2000 Idaho census is 1,293,953. How accurate is the regression line?

    c. What, specifically, does the number 11,130 have to say concerning Idaho's population?

ANSWERS

1. a. polynomial of degree 3
   b. not a polynomial because of division
   c. polynomial of degree 11
   d. not a polynomial because of x^{-3}
   e. not a polynomial beacuse of $\sqrt{x}$
   f. polynomial of degree 0—a constant polynomial

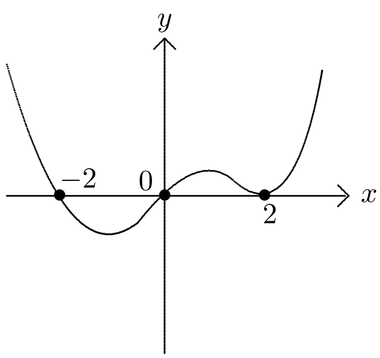
2. a. $p(x) = -\frac{1}{28} \cdot (2x+7) \cdot (x-2)^2$ b. $q(x) = a \cdot (x+2)^2 \cdot x \cdot (x-3)$ with $a < 0$

3. a. as $x \to +\infty$, $f(x) \to -\infty$ and as $x \to -\infty$, $f(x) \to +\infty$
   b. as $x \to +\infty$, $f(x) \to -\frac{2}{3}$ and as $x \to -\infty$, $f(x) \to -\frac{2}{3}$
   c. as $x \to +\infty$, $f(x) \to -\infty$ and as $x \to -\infty$, $f(x) \to -\infty$
   d. as $x \to +\infty$, $f(x) \to 0$ and as $x \to -\infty$, $f(x) \to 0$
   e. as $x \to +\infty$, $f(x) \to -12$ and as $x \to -\infty$, $f(x) \to -12$
   f. as $x \to +\infty$, $f(x) \to -\infty$ and as $x \to -\infty$, $f(x) \to +\infty$

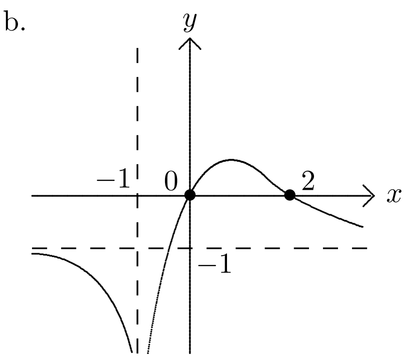
4. $y = \frac{1}{4}(x+6)^2 - 16$ so the vertex is $(-6, -16)$

5. $$f(x) = \begin{cases} \frac{2}{3}x + \frac{7}{3} & -2 < x \le 1 \\ -x + 4 & 1 < x < 2 \\ 3x - 4 & 2 < x \le 3 \end{cases}$$

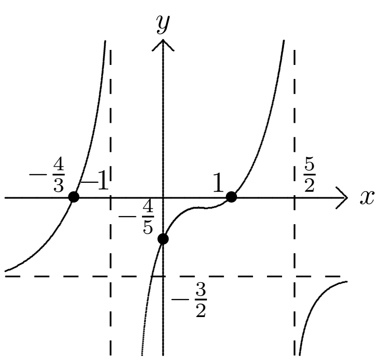
6. a.



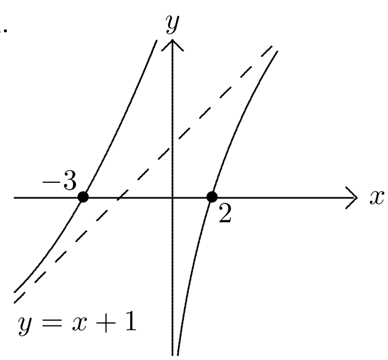
b.



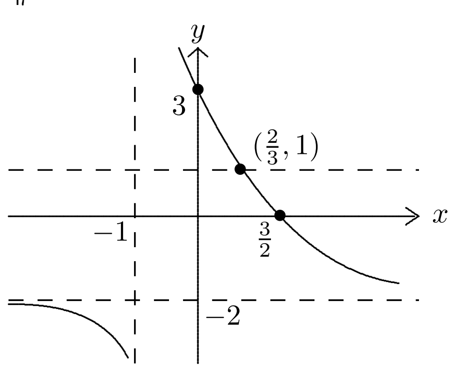
c.



d.



7. $(-\infty, -1) \cup [\frac{2}{3}, \infty)$



8. The distance from a typical point $(x, \sqrt{x})$ on the curve to the point $(3, 0)$ is

$$d = \sqrt{(x-3)^2 + (\sqrt{x} - 0)^2} = \sqrt{x^2 - 5\,x + 9}\,.$$

This will be a minimum at the vertex of the parabola $y = x^2 - 5\,x + 9$, i.e., at $x = \frac{5}{2}$.

9. Let the length of a side of the base of the box be x and let the height of the box be y. The volume $x^2 \cdot y$ must equal $1000\ in^3$. It follows that the total cost of making the box is

$$C = 2.00 + 0.05\,(x^2 + 4\,x\,y) + 0.07\,(x^2) = 2.00 + 0.12\,x^2 + \tfrac{200}{x}.$$

10. Let x represent the length of a side of the square pen and let y represent the length of the shorter side of the rectangular pen. The other side has length $2y$ so that the total area of the two pens is $A = x^2 + 2\,y^2$. We are also told to use $1000\ ft$ of fencing so that $1000 = 4x + 6y$. Thus $A = x^2 + 2\left(\frac{1000-4\,x}{6}\right)^2$.

11. a. $C(p) = 12{,}000 + 4 \cdot (2{,}000 - 25p) = 20{,}000 - 100p$
b. $R(p) = p \cdot (2{,}000 - 25p)$
c. $R(30) - C(30) = 30 \times 1{,}250 - 17{,}000 = 20{,}500$.
d. Since the profit function $R(p) - C(p) = 2{,}100p - 25\,p^2 - 20{,}000$ is quadratic, the maximum occurs at the vertex, i.e., when $p = 2{,}100/50 = \$42$.

12. Let $W(t)$ represent the waste produced in year t, measured in millions of tons. A linear model gives $W(t) = 82.3 + 2.84(t - 1960)$. The slope $2.84 = (139.1 - 82.3)/(1980 - 1960)$ represents the rate at which the waste is increasing, i.e., 2.84 million tons per year. The linear model gives $W(1990) = 167.5$ and $W(2000) = 195.9$ millions of tons.

13. a. 1,226,300
b. Regression line predicts 1,115,000. This is low by 178,953 or 14%.
c. As a rate of change, the number 11,130 is saying that, on average, the population of Idaho has been growing at a rate of 11,130 people per year.