MATH 143 — FINAL EXAM

Monday, December 16, 2002

3:00 - 5:00 pm

Name: _____

Section: _____

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Instructions:

1. No notes or formula sheets are allowed. A calculator will be necessary.

2. Unless a problem can be done in a single step, enough **work must be shown** to demonstrate proper understanding. A correct but less than obvious answer with no work shown is unlikely to be worth much in terms of credit.

3. All solutions should be written on the exam itself. Any scratch paper is to be collected by the instructor for disposal.

4. An **exact answer** is an expression, such as $3\,e^2 - \sqrt{7}$, that could be computed on a calculator. All answers should be exact, unless an approximate decimal value is asked for.

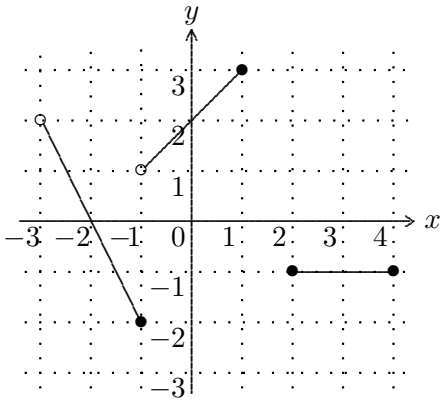
5. Problems are printed on both sides of the paper. Each page is worth 30 points, giving a total of 150 points.

1. (10 pts.) Determine the domain of the function $f(x) = \sqrt{\dfrac{x\,(1-2\,x)}{3\,x+7}}$. Express your answer using interval notation.

2. (10 pts.) Let $f(x) = x^2 + 3x$ and let $g(x) = 2x - 1$. Evaluate and simplify the following:
   a. $f(x-1) - g(x^2)$

   b. $(g \circ f)(-x)$

3. (10 pts.) The graph of a function $y = f(x)$ is given below. Use information in the graph to answer parts a through g. Give exact answers where possible, approximations when necessary.



   a. Domain $(f) =$

   b. Range $(f) =$

   c. $f(0) - f(2) =$

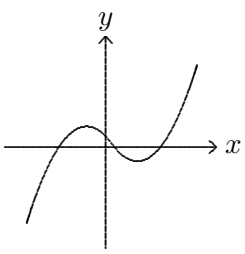
   d. $f(f(3)) =$

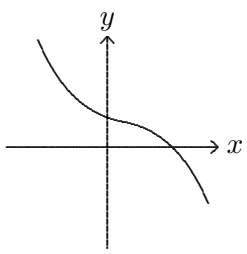
   e. Where is $f(x)$ increasing?

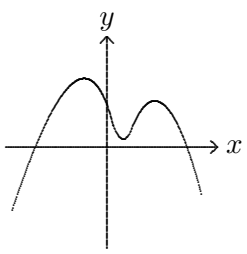
   f. Solve $f(x) = -1$.

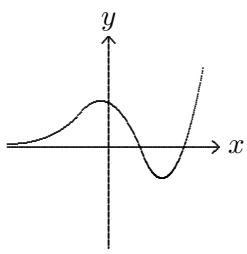
   g. Solve $f(x) > 2$.

4. (8 pts.) Which of the following graphs could be the graph of a degree 3 polynomial function? Indicate yes or no. If not, indicate why not. Assume that all essential features are given.

(a) 

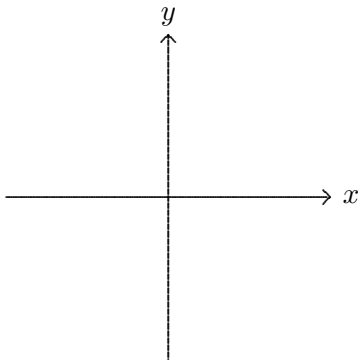
(b) 

(c) 

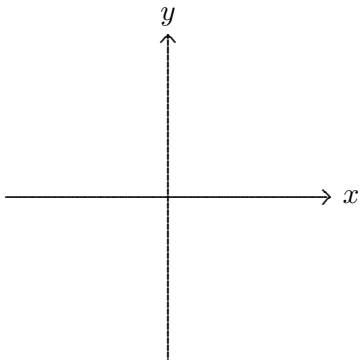
(d) 

5. (12 pts.) Give detailed graphs of the following functions. Be sure to determine exactly all intercepts, asymptotes, and vertices.

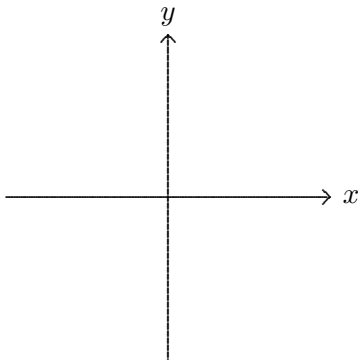
(a) $y = 6 - 2\,(x+1)^2$



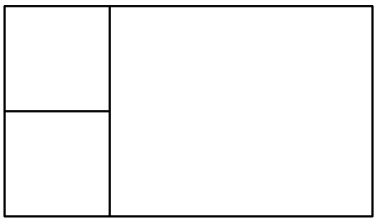
(b) $y = -\ln(x+2)$



(c) $y = \frac{3}{7-x}$



6. (10 pts.) A farmer wants to enclose two identical square pens and a larger, adjacent, rectangular pen, as in the figure. If he uses $2800\ ft$ of fence, what dimensions will maximize the total area of the three pens, and what will be that maximum area?



7. (8 pts.) For each of the following functions, describe the behavior of the function as $x \to +\infty$ and as $x \to -\infty$ by one of the following: (i) $f(x) \to +\infty$ (ii) $f(x) \to -\infty$ (iii) $f(x) \to L$ where L is a particular real number. Indicate which by entering the actual value of L or by entering $+\infty$ or $-\infty$ in the corresponding cell of the table.

   a. $f(x) = 83 - 47\,x^2 - x^3$

   b. $f(x) = \dfrac{2\,x^3 - 14}{67 - x}$

   c. $f(x) = \dfrac{(1-x)(2\,x+3)}{(x+4)(5-7x)}$

   d. $f(x) = 3 - 2\,e^{-5\,x}$

	$x \to -\infty$	$x \to +\infty$
a.		
b.		
c.		
d.		

8. (10 pts.) Solve $\log_{10}(x+3) + \log_{10}(5 - 6\,x) = 1$ algebraically. Show your work.

9. (12 pts.) The worldwide California condor population grew from 27 in 1985 to 63 in 1992.

   a. Estimate the condor population in 2002, using a linear model for the growth law.

   b. Estimate the condor population in 2002, using an exponential growth model.

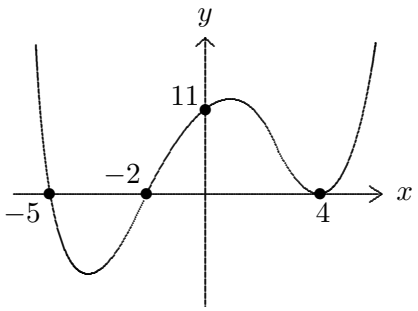
   c. Given that the current (2002) worldwide condor population is 200, including 46 at the World Center for Birds of Prey in Boise, does either model seem reasonable. Explain.

10. (10 pts.) The half life of a radioactive material is 5 years. How much of the material must one have right now in order to be sure to have $6\ \mu g$ of material 3 years from now? Compute to three decimal digits.

11. (10 pts.) The function $f(x) = \ln\left(\frac{x}{2\,x+1}\right)$ is one-to-one. Determine the inverse function f^{-1}.

12. (10 pts.) One root of the equation $7\,x^3 - 10\,x^2 + 27\,x + 20 = 0$ is $1 + 2\,i$. Determine the remaining roots.

13. (10 pts.) Give a possible formula for a polynomial with the following graph.



14. (8 pts.) Compute the quotient and remainder when $3\,x^3 + x^2 - 2$ is divided by $2\,x^2 + 3\,x - 5$.

15. (12 pts.) Let $f(x) = \frac{p(x)}{d(x)}$, where $p(x) = (x+3)(x-2)^2$ and $d(x) = (x-1)(2\,x+3)$. If the polynomial $p(x)$ is divided by $d(x)$ then the quotient is $q(x) = \frac{1}{2}\,x - \frac{3}{4}$ while the remainder is $r(x) = -\frac{23}{4}\,x + \frac{39}{4}$. Use this information to draw a detailed graph of $y = f(x)$.

Determine:

a. intercepts,

b. asymptotes.