

MATH 143 — FINAL EXAM

Monday, December 17, 2001

3:00 - 5:00 pm

Name: \_\_\_\_\_

Section: \_\_\_\_\_

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**Instructions:**

1. No notes or formula sheets are allowed. A calculator will be necessary. Any scratch paper must be turned in with the exam.
2. Unless a problem can be done in a single step, enough **work must be shown** to demonstrate proper understanding.
3. An **exact answer** is an expression, such as  $3e^2 - \sqrt{7}$ , that could be computed on a calculator. All answers should be exact, unless an approximate decimal value is asked for.
4. Problems are printed on both sides of the paper. Each page is worth 30 points, giving a total of 150 points.

1. (4 pts.) What is the vertex of the parabola  $y = 1.23(x + 4.56)^2 + 7.89$ ?

2. (6 pts.) Determine the domain of the function  $f(x) = \sqrt{\frac{x+3}{2x-1}}$ .

3. (8 pts.) Let  $f(x) = x^2 - 3x$  and let  $g(x) = 2x + 1$ . Evaluate and simplify the following:

a.  $f(1 - x)$

b.  $(g \circ f)(x^2)$

4. (12 pts.) The graph of a function  $y = f(x)$  is given below. Use information in the graph to answer parts a through g. Give exact answers where possible, approximations when necessary.

a. Domain ( $f$ ) =

b. Range ( $f$ ) =

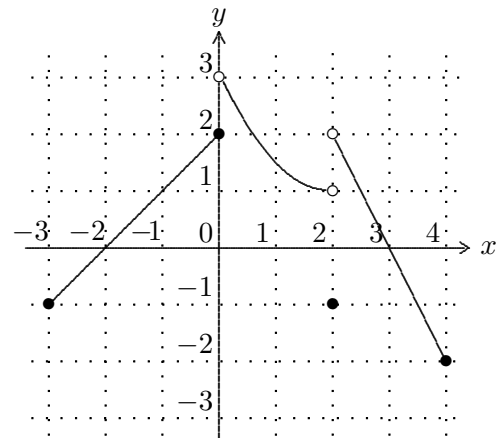
c.  $f(0) =$

d.  $f(f(2)) =$

e. Where is  $f(x)$  positive?

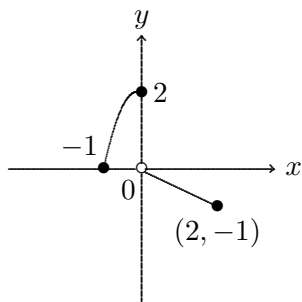
f. Where is  $f(x)$  increasing?

g. Solve  $f(x) = 1$ .

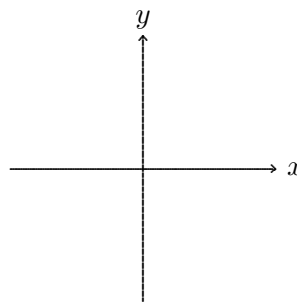


5. (8 pts.) The graph of a function  $f$  is given below as part a. Use it to draw the graphs in parts b and c. Indicate clearly the locations of the three points corresponding to the points marked with  $\bullet$  in the original graph.

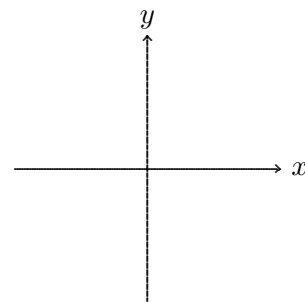
a.  $y = f(x)$



b.  $y = 1 + f(-x)$



c.  $y = f^{-1}(x)$



6. (12 pts.) For each of the following functions, describe the behavior of the function as  $x \rightarrow +\infty$  and as  $x \rightarrow -\infty$  by one of the following: (i)  $f(x) \rightarrow +\infty$  (ii)  $f(x) \rightarrow -\infty$  (iii)  $f(x) \rightarrow L$  where  $L$  is a definite real number. Indicate which by entering the number  $L$  or  $+\infty$  or  $-\infty$  in the appropriate cell of the table.

a.  $f(x) = 7x^3 - 2x^4 + 5$

b.  $f(x) = \frac{3x^3}{1 - 2x^2}$

c.  $f(x) = 3e^{-2x} - 4$

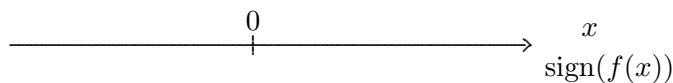
	$x \rightarrow -\infty$	$x \rightarrow +\infty$
a.		
b.		
c.		

7. (10 pts.) Let  $f(x) = \frac{x(1-x)}{(x+2)^2}$ .

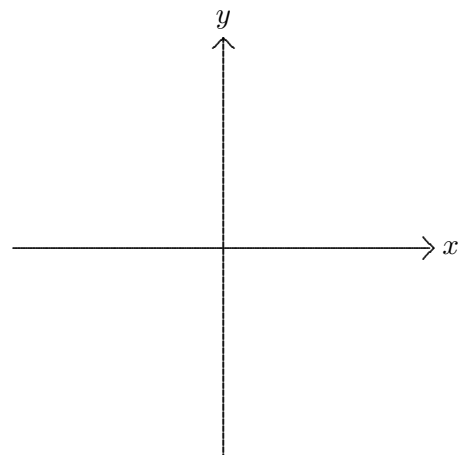
- a. Determine all intercepts of the graph  $y = f(x)$ .

- b. Determine all asymptotes of the graph  $y = f(x)$ .

- c. Exhibit the sign of  $f(x)$  on the number line:



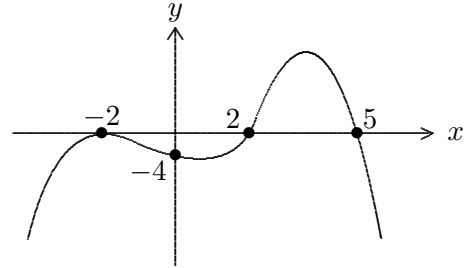
- d. Sketch the graph  $y = f(x)$ .



8. (6 pts.) What is the domain of the function  $f(x) = 2 \ln(3 - 7x) + 5$ ?
9. (8 pts.) Let  $f(x) = 3 + 2e^{-x}$ . Give an explicit formula for the inverse function  $f^{-1}$ .
10. (8 pts.) Solve  $5e^x = 2^{3-x}$ . Give exact answers along with three-digit decimal values.
11. (8 pts.) Solve the equation  $\ln(2x + 1) = \ln(x) + 3$  exactly.

12. (8 pts.) Find the point on the line  $y = x + 1$  that is closest to the point  $(6, 0)$ .
13. (12 pts.) A local wolf population has grown from 32 to 68 in 4 years. Determine the population after 7 additional years, assuming that the growth law is
- Linear
  - Exponential
14. (10 pts.) Suppose \$10,000 is deposited in an account that is earning 7% annual interest, compounded monthly. How long will it be before the account is worth \$15,000? Give an exact answer along with a three-digit decimal approximation.

15. (8 pts.) Give a possible formula for a polynomial with the following graph:



16. (6 pts.) Find a real quadratic function with a root at  $5 + 2i$ . Simplify.
17. (8 pts.) When a polynomial  $p(x)$  is divided by  $(3x^2 - 2)$  the quotient is  $(2x + 1)$  and the remainder is  $(5 - x)$ . What is  $p(x)$ ? Simplify your answer.
18. (8 pts.) The graph of the rational function  $f(x) = \frac{3x^2 - x}{2x + 1}$  has a slant asymptote. Determine the equation of that slant asymptote.