Homework Assignments 8 & 9

Modern Geometry, Math 343, Kriloff

Homework 8

(1) Complete the property table for the rigid motions.

(2) Prove Proposition 3: Let A, B, A', and B' be points in $\mathbb{R}^2$. There are at most two isometries that map A to A' and B to B'.

(3) Give the symbols that describe the following inverses (no justification required):

$$T_{\overrightarrow{AB}}^{-1}, \quad M_\ell^{-1}, \quad R_{P,\theta}^{-1}, \quad G_{\overrightarrow{AB}}^{-1}, \quad I^{-1}.$$

(4) Let $\triangle ABC$ and $\triangle A'B'C'$ be given and assume $\triangle ABC \simeq \triangle A'B'C'$.

(a) Prove there exists a reflection M_ℓ such that $M_\ell : A \mapsto A'$.

(b) Prove there exists a reflection M_m such that $M_m : A' \mapsto A'$ and $M_m : M_\ell(B) \mapsto B'$.

(c) Let $C_1 = M_m M_\ell(C)$. Prove $C' = C_1$ or $C' = M_{\overleftrightarrow{A'B'}}(C_1)$.

Homework 9

(1) Complete a sketch to illustrate investigation 8.2.2 (on combining reflections in parallel lines), give your conjecture and then provide a proof of your conjecture.

(2) Let A, B, A', and B' be points in $\mathbb{R}^2$.

(a) Prove that there is an isometry that maps A to A' and B to B' if and only if the length $AB = A'B'$.

(b) If the length $AB = A'B'$, prove there are exactly two isometries that map A to A' and B to B'. (Notice that this is a sharper form of Proposition 3.)

(3) Let $\triangle ABC$ and $\triangle A'B'C'$ be triangles in $\mathbb{R}^2$. Prove that $\triangle ABC \simeq \triangle A'B'C'$ if and only if there is an isometry mapping A to A', B to B', and C to C'.

(4) Let ℓ and m be perpendicular lines in $\mathbb{R}^2$.

(a) Determine all isometries F such that $F(\ell) = \ell$ and $F(m) = m$.

(b) Determine all isometries F such that $F(\ell) = m$ and $F(m) = \ell$.