Calculus I, Math 170, Kriloff
Final

Show **all** work completely on the pages below for full credit. **Simplify** when possible. Use **complete sentences** and **correct notation** throughout. When finished, **check** your work.

1. (14 points) Use **derivative rules** to find $f'(x)$. Answers need **not** be simplified.

   (a) $f(x) = 3\cos x \sin^2 x$

   (b) $f(x) = \dfrac{\sqrt[3]{x}}{ax^5 + b}$ (Let a and b be constant and give an answer in terms of a and b.)

2. (6 points) The position of a particle at time t, measured in seconds, is given by the function $s(t) = t^4 - 4t^2 + 1$ where s is measured in centimeters. Find the acceleration of the particle at $t = 2$ seconds.

3. (7 points) Use the **definition** of the derivative to find $f'(a)$ if $f(x) = 3x^2$.

4. (11 points) Use **implicit differentiation** to find the equation of the tangent line to the curve $y^4 - x^3y = 11 + x$ at the point $(-2, 1)$.

5. Each page of a book will have $30\,\text{in}^2$ of print, and each page must have 2 inch margins at the top and bottom and 1 inch margins at each side. What is the minimum possible area of such a page?

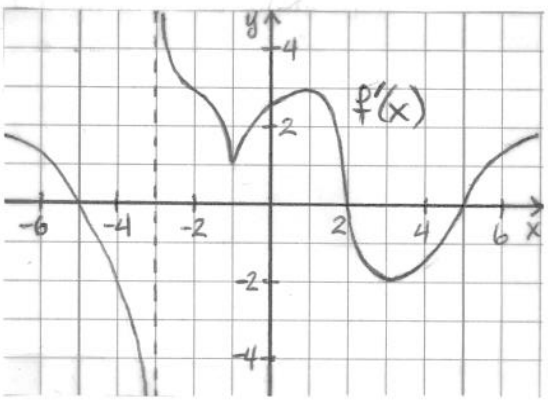
   (a) (2 points) Draw a diagram to illustrate the problem.

   (b) (3 points) Assign variables for important quantities. Mark them on the diagram **and clearly** state below **in words** what each represents.

   (c) (3 points) Express the variable you are to minimize in terms of the other variables.

   (d) (8 points) Express the variable you are to minimize in terms of only **one** other variable, **simplify**, and write the **domain** then **stop**. Do **not** finish solving the problem.

6. (9 points) The graph of the **derivative** $f'(x)$ of a function f is shown.



(a) On what interval(s) is f decreasing?

(b) Give all x value(s) where f has a local minimum.

(c) Give all x value(s) where f has an inflection point.

7. (a) (6 points) Use a precise, grammatically correct sentence to explain the meaning of the mathematical sentence $f'(5) = -2$ in terms of the graph of f. Do **not** use the word "derivative" in your answer.

(b) (3 points) Use a precise, grammatically correct sentence to explain what the mathematical sentence $f(3) = \lim_{x \to 3} f(x)$ tells us about f.

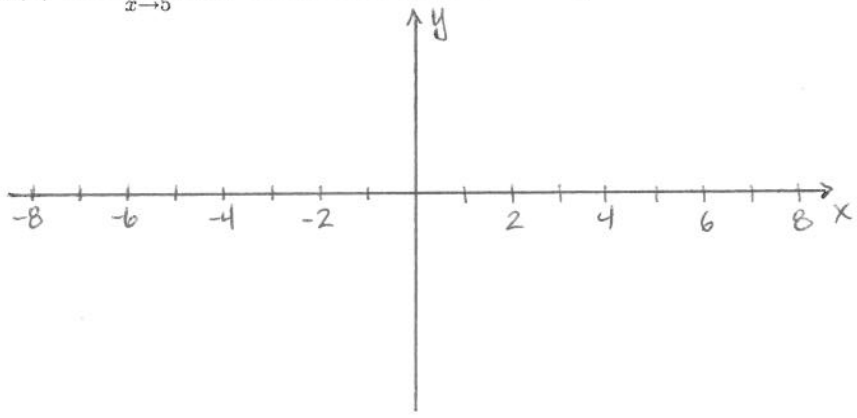
8. (8 points) Draw a careful graph of a function that has all of the following properties.

(a) $\lim_{x \to -3} f(x) = -\infty$

(b) f is defined but not continuous at $x = -1$

(c) f is continuous but not differentiable at $x = 2$

(d) $f(5)$ and $\lim_{x \to 5} f(x)$ both exist but are not equal.



9. Find the exact limit or state that the limit does not exist. To receive full credit you must **show work** or **explain** and give an **exact** answer.

(a) (4 points) $\lim_{x \to -3} \dfrac{4x + 12}{2x^2 + 5x - 3}$

(b) (7 points) $\lim_{x \to 16} \dfrac{x}{x - 16}$

10. The speed of a runner increased steadily during the first 12 seconds of a race, as shown in the table.

Time t (seconds)	0	3	6	9	12
Velocity $v(t)$ (ft/sec)	0	9.1	14.3	17.6	19.4

(a) (6 points) Give an overestimate for the total distance the runner traveled during the first 12 seconds of the race.

(b) (4 points) Explain how you know your answer to part (a) is an overestimate.

11. (23 points) Evaluate the following integrals or state that they do not exist. To receive full credit you must **show work** and give an **exact** answer.

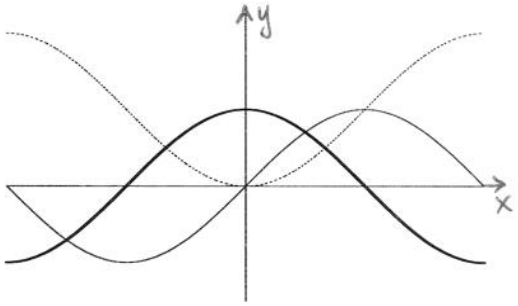
(a) $\int_0^4 (\sqrt{x}+1)^2\,dx$

(b) $\int \frac{x^3}{(2+x^4)^5}\,dx$

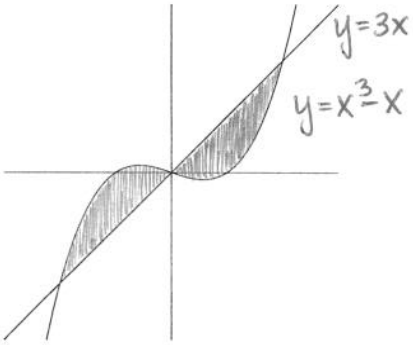
(c) $\int_0^{\pi/b} a\sin(bx)\,dx$ (Let a and b be constant and give an answer in terms of a and b.)

12. (6 points) Suppose the rate of change in the height of a plant in cm per week during its first 8 weeks of growth is modeled by $h'(t) = 6t^2 + 4t$ and the height after 5 weeks is 350 cm. Find the formula for the height of the plant after t weeks.

13. (6 points) Label each curve as one of $f(x)$, $f'(x)$, or $F(x) = \int_0^x f(t)dt$. No explanation required.



14. (8 points) Find the exact **area** of the shaded region bounded by $y = x^3 - x$ and $y = 3x$.



15. (6 points) Consider the region bounded by $y = x(x-3)^2$ and the x-axis (see the graph). Set up but **do not** evaluate an integral to calculate the volume of the solid formed by revolving this region about the y-axis.