Calculus I, Math 170, Kriloff
Exam 3

Show **all** work completely on the pages below for full credit. **Simplify** when possible. Use **complete sentences** and **correct notation** throughout. When finished, **check** your work.

1. (20 points) Find or evaluate the following integrals or state that they do not exist and explain why. Show all work and give an exact answer when appropriate for full credit.

(a) $\int (1+3\cos\theta)^5 \sin\theta \, d\theta$ $= \int u^5\left(-\frac{1}{3}du\right) = -\frac{1}{3}\int u^5 du$

Let $u = 1+3\cos\theta$

so $du = -3\sin\theta \, d\theta$

and $-\frac{1}{3}du = d\theta$

$= -\frac{1}{3}\frac{u^6}{6} + C$

$= -\frac{1}{18}u^6 + C$

$= -\frac{1}{18}(1+3\cos\theta)^6 + C$

(b) $\int_1^3 \frac{3x+1}{\sqrt{x}}\,dx$ $= \int_1^3 (3x^{1/2} + x^{-1/2})dx$

$= 3\frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2}\Big|_1^3 = 2x^{3/2} + 2x^{1/2}\Big|_1^3$

$= 2(\sqrt{27} + \sqrt{3} - 1 - 1) = 2(3\sqrt{3} + \sqrt{3} - 2) = 8\sqrt{3} - 4$

(c) $\int_0^2 \frac{1}{x^2-1}\,dx$ does not exist since $\frac{1}{x^2-1}$ has an infinite discontinuity at $x=1$ in $[0,2]$.

(Though remember, improper integrals might actually exist, but it requires using limits to check - see Section 8.8)

2. (6 points) If $\int_3^6 f(t)\,dt = -5$, find $\int_6^3 (4+2f(t))\,dt$ using properties of integrals.

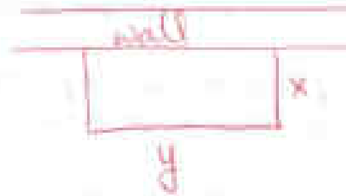
$\int_6^3 (4+2f(t))dt = -\int_3^6 (4+2f(t))dt$

$= -\int_3^6 4dt - 2\int_3^6 f(t)dt$

$= -4(6-3) - 2(-5) = -12 + 10 = -2$

3. A man wants to fence a rectangular area of 20 square feet adjacent to a wall. He will use fence that costs \$5 per foot for the two opposite sides and fence that costs \$6 per foot on the remaining side between them. Find the dimensions that will minimize the cost of the fence.

(a) (3 points) Draw a diagram illustrating the problem and mark choices of variables on the diagram.



(b) (5 points) State what is given and what you are to find, introducing and describing any variables needed to do so.

He uses $2x$ feet of fence that costs \$5/ft and y feet of fence that costs \$6/ft.
Area $A = 20\text{ ft}^2$.
We are to find x, y so that cost C is smallest.

(c) (3 points) Write an expression for the quantity to be optimized using the variables.

$$C = 5 \cdot 2x + 6 \cdot y$$
$$= 10x + 6y$$

(d) (2 points) Use the given information to write an equation that relates the variables.

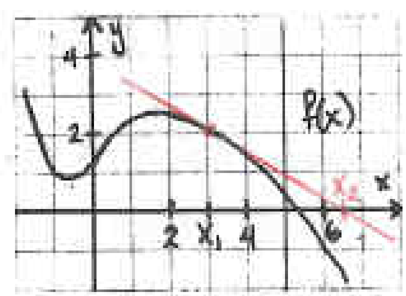
$A = xy = 20$ so $y = \frac{20}{x}$

(e) (4 points) Use part (d) to write the quantity to be optimized as a function of one variable and write the domain of this function then stop. Do not finish solving.

$$C = 10x + 6 \cdot \frac{20}{x}$$
$$= 10x + \frac{120}{x}$$

Domain: $x > 0$

4. (6 points) The graph of a function f and $x_1 = 3$, a first approximation to its root, are shown.



(a) Draw on the graph how to find the second approximation x_2 using Newton's method.
$x_2 \approx$ 6.5

(b) Was $x_1 = 3$ a good first choice to use in Newton's method? Why or why not?

Yes – if one continues the process, it appears that x_n will approach the root at approximately 5.3 (draw a couple more tangent lines).
(Notice that if $x_1 \approx 1.8$ or $x_1 \approx -0.8$ then Newton's method would fail since in $x_1 - f(x_1)/f'(x_1)$, $f'(x_1) = 0$.)

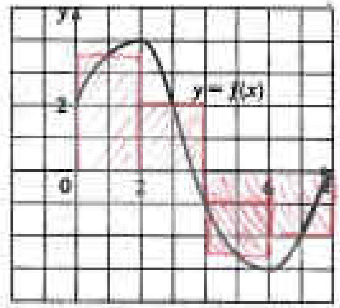
5. (9 points) By reading values from the graph, use 4 rectangles and midpoints to give an estimate for $\int_0^8 f(x)\,dx$. Also draw the approximating rectangles on the graph.

$$\int_0^8 f(x)\,dx \approx \sum_{i=1}^{4} f(\bar{x}_i)\Delta x$$

$$= (3.5+2-2.5-2)\cdot 2$$

$$= 1\cdot 2$$

$$= 2$$



6. (7 points) Write an expression for $\displaystyle\int_2^9 (\sqrt{x}+4x^3)\,dx$ as a limit of a Riemann sum using left endpoints. Do not attempt to evaluate the limit.

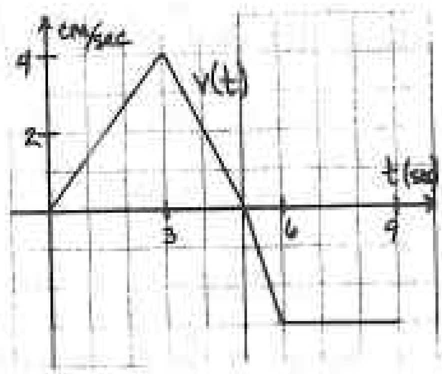
Substitute for any x_i that occur.

$$\int_2^9 (\sqrt{x}+4x^3)\,dx = \lim_{n\to\infty} \sum_{i=0}^{n-1} \left(\sqrt{2+\frac{7i}{n}} + 4\left(2+\frac{7i}{n}\right)^3\right)\cdot\frac{7}{n}$$

$$\Delta x = \frac{9-2}{n} = \frac{7}{n} \qquad \left(= \lim_{n\to\infty}\sum_{i=0}^{n-1} f(x_i)\Delta x\right)$$

$$x_i = 2 + i\cdot\frac{7}{n} = 2+\frac{7i}{n}$$

7. (11 points) A particle is moving with velocity $v(t)$ cm/sec, as shown in the graph.



using integrals

(a) Write an expression for the displacement of the particle during the first 9 seconds and find this displacement using the graph.

Displacement during the first 9 seconds is

$$\int_0^9 v(t)\,dt = \frac{1}{2}(5)4 - \frac{1}{2}\cdot 1\cdot 3 - 3\cdot 3$$

$$= 10 - \frac{3}{2} - 9 = -\frac{1}{2}\text{ cm}$$

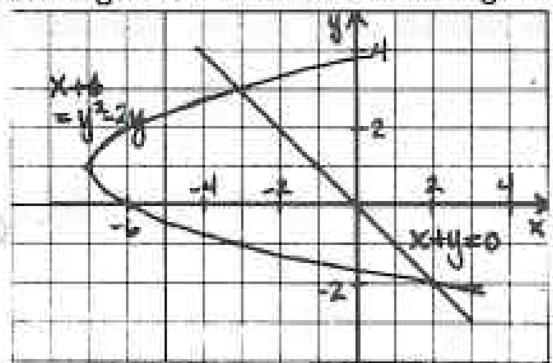
4 - 1 each area
1 for subtracting here, adding below

(b) Write an expression for the total distance travelled by the particle during the first 9 seconds and find this distance using the graph.

Distance travelled during the first 9 seconds is

$$\int_0^9 |v(t)|\,dt = 10 + \frac{3}{2} + 9 = \frac{41}{2}\text{ cm or } 20.5\text{ cm}$$

8. (7 points) Set up, but **do not evaluate** a definite integral to find the area of the region shown in the graph.



$x = y^2 - 2y - 6$ and $x = -y$

$\int_{-2}^{3} (-y - (y^2 - 2y - 6))\,dy$

$= \int_{-2}^{3} (-y^2 + y + 6)\,dy$

1 for order
1 for subtr
1 for integral
of dy

9. (12 points) For each of the following statements, state if it is true or false. If true, explain why, if false, **give a counterexample** to prove the statement is false. Assume f and g are continuous and all integrals exist.

(a) $\int_{-3}^{3} f(x)\,dx = 2\int_{0}^{3} f(x)\,dx$ False. If $f(x)$ is even this is true.

One easy counterexample is to take $f(x) = x$. Then $\int_{-3}^{3} x\,dx = 0$ since x is odd, but

$2\int_0^3 x\,dx = 2 \cdot \frac{x^2}{2}\Big|_0^3 = x^2\Big|_0^3 = 9 \neq 0$.

(b) $\int_a^b f(x)\,dx \int_a^b g(x)\,dx = \int_a^b [f(x)g(x)]\,dx$ False.

One possible counterexample is to take $a=0$, $b=1$, $f(x)=g(x)=x$.

$\int_0^1 x\,dx \int_0^1 x\,dx = \frac{x^2}{2}\Big|_0^1 \cdot \frac{x^2}{2}\Big|_0^1 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ but

4 bonus?

$\int_0^1 x \cdot x\,dx = \int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3} \neq \frac{1}{4}$.

(c) For $a > 0$, $\frac{d}{dx}\int_a^x \cos(\sqrt{t})\,dt = \cos(\sqrt{x})$

3 True. This is true by the First part of the Fundamental Theorem of Calculus with $f(t) = \cos\sqrt{t}$.

10. (5 points) Carefully and precisely state Part I of the Fundamental Theorem of Calculus.

If f is continuous on $[a,b]$, then the (net area) function $g(x) = \int_a^x f(t)\,dt$ is continuous on $[a,b]$, differentiable on (a,b), and satisfies $g'(x) = f(x)$.