Calculus I, Math 170, Kriloff
Exam 2

A correct answer with little or no supporting work will be worth very little, so show **all** work. **Simplify** and use **complete sentences** and **correct notation** throughout. When finished, **check** your work. Attach extra paper if needed.

1. (11 points) The position in centimeters of a mass on a spring after t seconds is given by $s(t) = 8\cos\left(\frac{\pi}{4}t\right)$. Find the exact acceleration at $t = 1$ second. Include units.

$v(t) = s'(t) = 8\left(-\sin\left(\frac{\pi}{4}t\right)\right)\frac{\pi}{4} = -2\pi\sin\left(\frac{\pi}{4}t\right)$ 3.5

$a(t) = s''(t) = -2\pi\cdot\cos\left(\frac{\pi}{4}t\right)\cdot\frac{\pi}{4} = -\frac{\pi^2}{2}\cos\left(\frac{\pi}{4}t\right)$ 4

$a(1) = -\frac{\pi^2}{2}\cdot\cos\left(\frac{\pi}{4}\right)$ 1

$= -\frac{\pi^2}{2}\cdot\frac{\sqrt{2}}{2}$ 1

$= -\frac{\sqrt{2}\,\pi^2}{4}$ cm/sec^2 (or $\frac{\text{cm/sec}}{\text{sec}}$) 1.5

2. (7 points) Find the **differential** of the function $y = (3x - \sec x)^4$.

$dy = 4(3x - \sec x)^3(3 - \sec x\tan x)\,dx$

(The formula for the differential is $dy = f'(x)\,dx$)

3. (9 points) Use implicit differentiation and solve for $\frac{dy}{dx}$ in $y^3 - xy = 10$.

$3y^2\frac{dy}{dx} - \left(1\cdot y + x\cdot\frac{dy}{dx}\right) = 0$

$3y^2\frac{dy}{dx} - y - x\frac{dy}{dx} = 0$

$\frac{dy}{dx}(3y^2 - x) = y$

$\frac{dy}{dx} = \frac{y}{3y^2 - x}$

4. (6 points) If $y = f(x)$, write the formula for the linear approximation to $f(x)$ at $x = a$.

$f(x) \approx f(a) + f'(a)(x-a)$

5. (20 points) Find the following or state that there are none. Assume that the domain of f is all real numbers and that its **derivative** is $f'(x) = \dfrac{x-2}{\sqrt[3]{x}}$. $= \dfrac{x}{x^{1/3}} - \dfrac{2}{x^{1/3}} = x^{2/3} - 2x^{-1/3}$

$f'(x) = 0$ at $x-2=0$ or $x=2$

$f'(x)$ does not exist at $\sqrt[3]{x}=0$ or $x=0$

	0		2	
$x-2$	$-$	$-$	$+$	
$\sqrt[3]{x}$	$-$	$+$	$+$	
$f'(x)$	$+$	$-$	$+$	

3

2 (a) Critical value(s) (or number(s)) of f: $0, 2$

3 (b) Interval(s) where f is increasing: $(-\infty, 0) \cup (2, \infty)$

2 (c) Interval(s) where f is decreasing: $(0,2)$

1 (d) Value(s) of x at which f has a local maximum. 0

1 (e) Value(s) of x at which f has a local minimum. 2

5 (f) $f''(x) = \frac{2}{3}x^{-1/3} + \frac{2}{3}x^{-4/3} = \frac{2}{3}\left(\frac{1}{x^{1/3}} + \frac{1}{x^{4/3}}\right) = \frac{2}{3}\left(\frac{x+1}{x^{4/3}}\right)$

(or use the quotient rule: $\dfrac{x^{1/3} - (x-2)\frac{1}{3}x^{-2/3}}{x^{2/3}} = \dfrac{\frac{3x}{3x^{2/3}} - \frac{(x-2)}{3x^{2/3}}}{x^{2/3}} = \dfrac{2x+2}{3x^{4/3}} = \dfrac{2}{3}\left(\dfrac{x+1}{x^{4/3}}\right)$.)

Since $x^{4/3} > 0$, the sign chart is just that for $x+1$:

	-1		0	
$f''(x)$	$-$	$+$	$+$	

2 (g) Interval(s) where f is concave down. $(-\infty, -1)$

1 (h) Value(s) of x at which f has an inflection point. -1

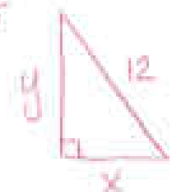
6. (6 points) Give a complete sentence stating the Extreme Value Theorem.

If f is continuous on $[a,b]$, then f takes on an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some c, d in $[a,b]$

7. (15 points) A ladder 12 feet long rests against a vertical wall. The base of the ladder begins to slide along the ground at a rate of 1.5 ft/sec. How fast is the top of the ladder descending when the base is 4 feet away from the wall? Include units. Start by completing the following.

Let x be the distance from the base of the ladder to the wall.

Let y be the height where the top of the ladder hits the wall.



Information given: $\frac{dx}{dt} = 1.5$ ft/sec, $x = 4$ ft

Information to find: $\frac{dy}{dt}$

$x^2 + y^2 = 12^2 = 144$

Differentiate both sides with respect to t: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

When $x = 4$, $y = \sqrt{12^2 - x^2} = \sqrt{144 - 16} = \sqrt{128} = 8\sqrt{2}$

$4(1.5) + 8\sqrt{2}\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-6}{8\sqrt{2}} = \frac{-3}{4\sqrt{2}}$ ft/sec $\approx -.53$ ft/sec.

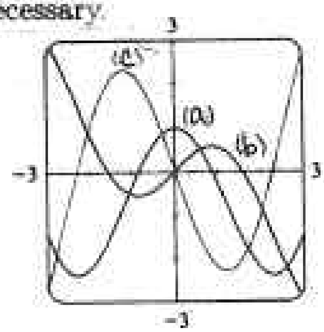
8. (9 points) Calculate the exact limit. Show steps carefully.

$$\lim_{x\to-\infty} \frac{4x}{-x+\sqrt{3x^2+1}} \cdot \frac{1/x}{1/x} = \lim_{x\to-\infty} \frac{4x/x}{-\frac{x}{x} + \sqrt{3x^2+1}\cdot\frac{-1}{\sqrt{x^2}}}$$

$$= \lim_{x\to-\infty} \frac{4}{-1-\sqrt{3+1/x^2}} = \frac{4}{-1-\sqrt{3}} \text{ or } \frac{-4}{1+\sqrt{3}}$$

$$\left(\text{or } \frac{-4}{1+\sqrt{3}}\cdot\frac{1-\sqrt{3}}{1-\sqrt{3}} = \frac{-4(1-\sqrt{3})}{-2} = 2(1-\sqrt{3})\right)$$

BONUS 9. (4 points) Label each graph as f, f', or f''. No explanation necessary.

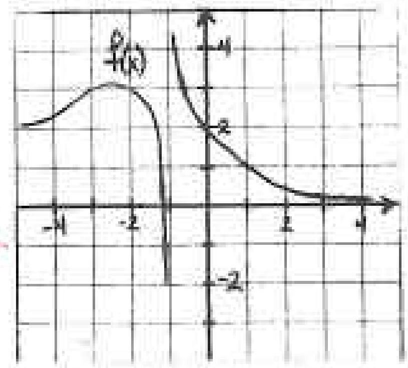


(a) f'

(b) f

(c) f''

10. (8 points) The graph of the function f is given.



(a) Write the equations of the asymptotes.

$x=-1$ is the vertical asymptote

$y=2$
$y=0$ are the horizontal asymptotes.

(b) Write four mathematical equations involving limits to describe the behavior of f near all of its asymptotes.

$\lim_{x\to-\infty} f(x)=2$ $\quad$ $\lim_{x\to-1^-} f(x)=-\infty$

$\lim_{x\to\infty} f(x)=0$ $\quad$ $\lim_{x\to-1^+} f(x)=\infty$

11. (9 points) State whether the following are **always**, **sometimes**, or **never** true. No explanation required. Each part is 3 points, all or nothing.

(a) If f is differentiable, then $\dfrac{d}{dx}f(\sqrt{x}) = \dfrac{f'(\sqrt{x})}{2\sqrt{x}}$.

Always (This is the correct chain rule result.)

(b) If $f''(4) = 0$, then $(4, f(4))$ is an inflection point of the curve $y = f(x)$.

Sometimes (If the sign of $f''(x)$ also changes at 4.)

(c) If $f(x) = x^n$, where n is a positive integer, then $f^{(n+1)}(x) = 0$.

Always