Calculus I, Math 170, Kriloff
Exam 1

A correct answer with little or no supporting work will be worth very little, so show **all** work. **Simplify** and use **complete sentences** and **correct notation** throughout. When finished, **check** your work.

1. (8 points) Find the equation of the tangent line to $f(x) = \sqrt[3]{x^2} - \dfrac{8}{\sqrt[6]{x}}$ at $x = 1$.

2. (16 points) Use derivative formulas to find y' in terms of x or f.

   (a) $y = \dfrac{(x-2)(3x+5)}{x+\sqrt{7}}$

   (b) $y = [f(x)]^2 = f(x)f(x)$

Replace with questions from 2.6

3. (4 points) Ohm's Law relating resistance R, voltage V, and current I, is $R = V/I$. Assuming resistance is constant, find the rate of change of voltage with respect to current.

4. (8 points) Find $f'(x)$ using the definition of the derivative if $f(x) = \dfrac{4}{3-2x}$.

5. (18 points) Find the following limits. Explain your reasoning clearly.

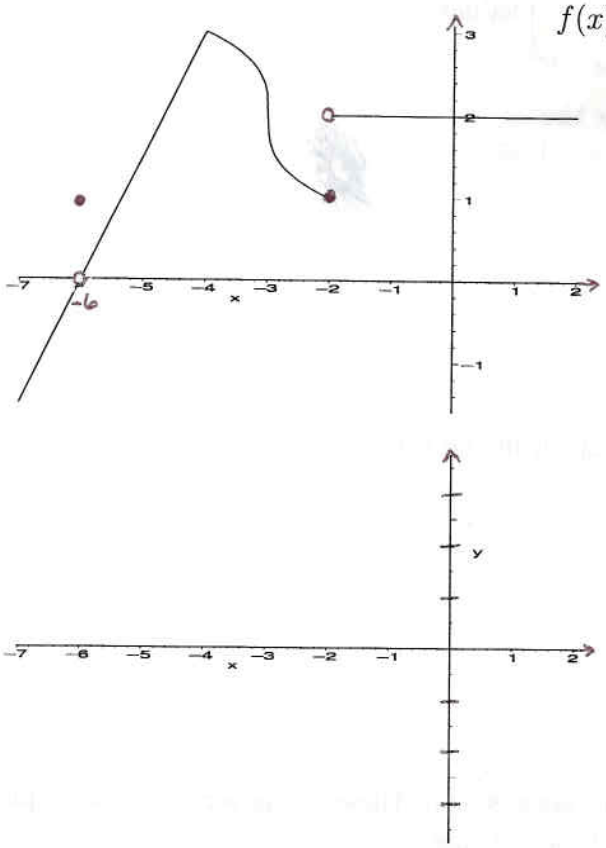
   (a) $\lim\limits_{t\to -6} \dfrac{2t+12}{t^2+3t-18}$

   (b) $\lim\limits_{x\to 5^-} \dfrac{2x}{x-5}$

   (c) $\lim\limits_{x\to 1} f(x)$ where $f(x) = \begin{cases} 2+x, & \text{if } x < 1; \\ 2/x, & \text{if } x \geq 1 \end{cases}$

6. (4 points) Give a complete sentence stating a precise definition of continuity.

7. (22 points) Below is the graph of f.



$$f(x) = \begin{cases} \frac{3}{2}x + 9 & \text{if } x < -6 \text{ or } -6 < x < -4 \\ 1 & \text{if } x = -6 \\ ? & \text{if } -4 \leq x \leq -2 \\ 2 & \text{if } -2 < x \end{cases}$$

(a) Suppose that on the interval $-4 \leq x \leq -2$, $f(x)$ is given by translating the graph of $g(x) = -x^{1/3}$ left 3 units and up 2. Find the formula to replace ? in $f(x)$.

(b) Carefully sketch the graph of $f'(x)$ on the axes above. Mark the y-axis scale.

(c) Fill in the table with all values a so that f is not differentiable at $x = a$ and give a **thorough reason** for each.

At	the function f is not differentiable because

8. (14 points) The interest rate on U.S. treasury bills is a function of time. The table gives midyear values of this function $I(t)$ over a nine-year period.

t	$I(t)$	t	$I(t)$
1983	8.62	1988	6.67
1984	9.57	1989	8.11
1985	7.49	1990	7.51
1986	5.97	1991	5.41
1987	5.83	1992	3.46

(a) Give a complete sentence describing the meaning of the statement $I'(1990) = -1.35$.

(b) Estimate $I'(1988)$ using the values given in the table.

9. (9 points) State whether the following are always, sometimes, or never true. No explanation required. Each part is 3 points, all or nothing.

(a) If $f'(r)$ exists, then $\lim_{x \to r} f(x) = f(r)$.

(b) $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ do not exist, but $\lim_{x \to a}[f(x) + g(x)]$ does exist.

(c) If $f(a) = 4$, $f(b) = -7$ and f is not continuous on $[a, b]$, then there is no value c between a and b so that $f(c) = 0$.

Optional bonus question. (5 points) Give a grammatically correct sentence stating the complete precise definition of the statement $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$. (This is proven in Section 3.5.)