

Name \_\_\_\_\_

Instructor \_\_\_\_\_

Section number \_\_\_\_\_

**Directions:** Show all work and simplify all answers. If you use your calculator to arrive at a conclusion, state what you did on your calculator. Give exact answers unless asked for a decimal approximation. Box your final answers. There are 200 points possible: budget your time on each problem effectively.

NPC means “no partial credit”.

Page	Points possible	Points earned
2	28	
3	28	
4	30	
5	22	
6	20	
7	22	
8	27	
9	23	
TOTAL	200	

1. [7 × 4 = 28 points]

(a) [4 points, NPC] Let  $f(x) = \sqrt{3 - x^2}$ . Find  $f(1)$ .

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(b) [4 points, NPC] Let  $f(x) = x - 1$ . Find  $(f \circ f)(x)$ . Simplify your answer.

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(c) [4 points, NPC] Fill in the blank with one of the options (i), (ii), or (iii). A polynomial of degree five \_\_\_\_\_ has four turning points.

- i. always
- ii. sometimes
- iii. never

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(d) [4 points, NPC] Find the domain of  $g(x) = 100^{x-3}$ .

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(e) [4 points, NPC] Solve for  $x$  exactly:  $\ln x = -3$ .

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(f) [4 points, NPC] Simplify:  $e^{2 \ln x}$ .

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(g) [4 points, NPC] Assume that  $b > 0$ . Simplify  $\log_b(1/b^2)$ .

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2. [7 × 4 = 28 points]

(a) [4 points, NPC] Simplify  $\log_3 3^{2x-1}$ .

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(b) [4 points, NPC] Find the range of  $H(x) = \ln(x - 1)$ .

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(c) [4 points, NPC] Give the exact equation for the vertical asymptote to the graph  $y = \frac{5x - 6}{9x + 7}$ .

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(d) [4 points, NPC] Give the exact equation for the horizontal asymptote to the graph  $y = \frac{5x - 6}{9x + 7}$ .

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(e) [4 points, NPC] A polynomial  $p(x)$  with real coefficients has  $3 - 2i$  as a complex root. What other complex root is the polynomial  $p$  guaranteed to have?

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(f) [4 points, NPC] Write  $(3 + i)^2$  in the form  $a + bi$ .

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(g) [4 points, NPC] Write  $\frac{1}{i}$  in the form  $a + bi$ .

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3. [15 points] Let  $f(x) = \sqrt{9 - x^2}$ .

(a) [8 points] Find the domain of  $f$ . Express your answer in interval notation.

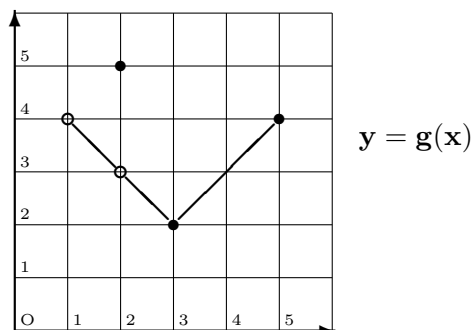
(b) [7 points] Express  $f$  as the composition  $g \circ h$  of simpler functions  $g$  and  $h$ .

$$g(x) = \underline{\hspace{10em}}$$

$$h(x) = \underline{\hspace{10em}}$$

4. [15 points] Let  $f(x) = x^2 - 3$ . Find  $\frac{f(x+h) - f(x)}{h}$ . Simplify your answer.

5. [11 points] The graph of  $y = g(x)$  is given.

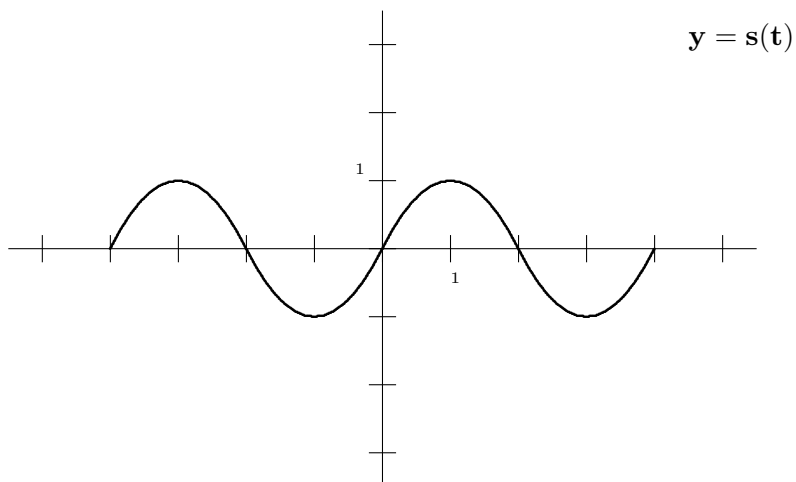


- (a) [4 points] Find the domain of  $g$ .

- (b) [4 points] Find the range of  $g$ .

- (c) [3 points] Find all values of  $x$  so that  $g(x) = 3$ .

6. [11 points] The graph of  $y = s(t)$  is given. Graph  $y = -s(t - \frac{1}{2}) + 2$  on the same axes.



7. [10 points] Let  $x$  denote a temperature on the Celsius scale and let  $y$  denote the corresponding temperature on the Fahrenheit scale. Find a linear function relating  $x$  and  $y$ . Use the facts that  $32^\circ$  F corresponds to  $0^\circ$  C and  $212^\circ$  F corresponds to  $100^\circ$  C. Write the function in the form  $y = Ax + B$ .

8. [10 points] The perimeter of a rectangle is 200 cm. Express the length of the diagonal as a function of the width  $x$ .

9. [12 points] Suppose that the revenue generated by selling  $x$  units of a certain commodity is given by  $R(x) = -\frac{1}{5}x^2 + 200x$ . Assume that  $R$  is in dollars. What is the maximum revenue possible? Completely justify your answer using algebra; do not use your graphing calculator.

10. [10 points] Let  $g(x) = 2x^3 - 1$ . Find a formula for  $g^{-1}(x)$ .

11. [14 points] Write the quantity using sums and differences of simpler logarithmic expressions. Express the answer so that logarithms of products, quotients, and powers do not appear. Simplify your answer.

$$\ln\left(\frac{x^2}{e\sqrt{x+4}}\right)$$

12. [13 points] At the start of an experiment, a colony has an initial population of 3000. After two hours, there are 3500 bacteria present. When will the population reach 10,000? (Round your answer to the nearest unit.)

13. [13 points] Solve:  $\log_3 x + \log_3(x + 2) = 1$ .

14. [10 points] Solve the equation  $x^3 + x^2 - 11x - 15 = 0$  given that one of the roots is  $-3$ .