

Directions: Show all work and simplify all answers. If you use your calculator to arrive at a conclusion, state what you did on your calculator. Give exact answers unless asked for a decimal approximation. Box your final answers. There are 200 points possible: budget your time on each problem effectively.

NPC means “no partial credit”.

1. $[7 \times 4 = 28 \text{ points}]$

- (a) [4 points, NPC] Let $f(x) = \sqrt{3 - x^2}$. Find $f(1)$.
- (b) [4 points, NPC] Let $f(x) = x - 1$. Find $(f \circ f)(x)$. Simplify your answer.
- (c) [4 points, NPC] Fill in the blank with one of the options (i), (ii), or (iii). A polynomial of degree five _____ has four turning points.
- i. always
 - ii. sometimes
 - iii. never
- (d) [4 points, NPC] Find the domain of $g(x) = 100^{x-3}$.
- (e) [4 points, NPC] Solve for x exactly: $\ln x = -3$.
- (f) [4 points, NPC] Simplify: $e^{2 \ln x}$.
- (g) [4 points, NPC] Assume that $b > 0$. Simplify $\log_b(1/b^2)$.

2. $[7 \times 4 = 28 \text{ points}]$

- (a) [4 points, NPC] Simplify $\log_3 3^{2x-1}$.
- (b) [4 points, NPC] Find the range of $H(x) = \ln(x - 1)$.
- (c) [4 points, NPC] Give the exact equation for the vertical asymptote to the graph $y = \frac{5x - 6}{9x + 7}$.
- (d) [4 points, NPC] Give the exact equation for the horizontal asymptote to the graph $y = \frac{5x - 6}{9x + 7}$.

(e) [4 points, NPC] A polynomial $p(x)$ with real coefficients has $3 - 2i$ as a complex root. What other complex root is the polynomial p guaranteed to have?

(f) [4 points, NPC] Write $(3 + i)^2$ in the form $a + bi$.

(g) [4 points, NPC] Write $\frac{1}{i}$ in the form $a + bi$.

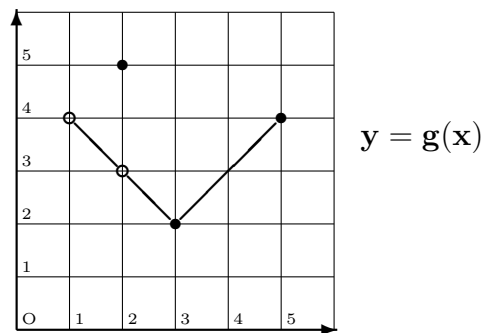
3. [15 points] Let $f(x) = \sqrt{9 - x^2}$.

(a) [8 points] Find the domain of f . Express your answer in interval notation.

(b) [7 points] Express f as the composition $g \circ h$ of simpler functions g and h .

4. [15 points] Let $f(x) = x^2 - 3$. Find $\frac{f(x+h) - f(x)}{h}$. Simplify your answer.

5. [11 points] The graph of $y = g(x)$ is given.

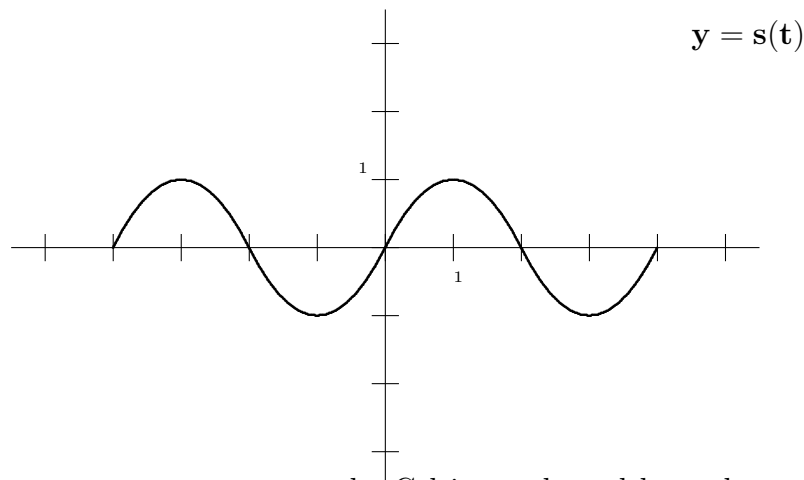


(a) [4 points] Find the domain of g .

(b) [4 points] Find the range of g .

(c) [3 points] Find all values of x so that $g(x) = 3$.

6. [11 points] The graph of $y = s(t)$ is given. Graph $y = -s(t - \frac{1}{2}) + 2$ on the same axes.



7. [10 points] Let x denote a temperature on the Celsius scale and let y denote the corresponding temperature on the Fahrenheit scale. Find a linear function relating x and y . Use the facts that 32° F corresponds to 0° C and 212° F corresponds to 100° C. Write the function in the form $y = Ax + B$.
8. [10 points] The perimeter of a rectangle is 200 cm. Express the length of the diagonal as a function of the width x .
9. [12 points] Suppose that the revenue generated by selling x units of a certain commodity is given by $R(x) = -\frac{1}{5}x^2 + 200x$. Assume that R is in dollars. What is the maximum revenue possible? Completely justify your answer using algebra; do not use your graphing calculator.
10. [10 points] Let $g(x) = 2x^3 - 1$. Find a formula for $g^{-1}(x)$.
11. [14 points] Write the quantity using sums and differences of simpler logarithmic expressions. Express the answer so that logarithms of products, quotients, and powers do not appear. Simplify your answer.
- $$\ln \left(\frac{x^2}{e\sqrt{x+4}} \right)$$
12. [13 points] At the start of an experiment, a colony has an initial population of 3000. After two hours, there are 3500 bacteria present. When will the population reach 10,000? (Round your answer to the nearest unit.)
13. [13 points] Solve: $\log_3 x + \log_3(x + 2) = 1$.
14. [10 points] Solve the equation $x^3 + x^2 - 11x - 15 = 0$ given that one of the roots is -3 .