College Algebra, Math 143, Kriloff
Exam 3, Fall 2002

Instructions: Show your work! If you use a calculator, write what you computed or sketch the graph you used. Include intermediate steps. Multiple answers or a correct answer without a reasonable amount of work might receive no credit. Give exact answers unless asked for decimal approximations. When finished, check your work by hand or on your calculator.

Compare to 5.1 #11-12.

1. (6 points) Use properties of exponents to solve for y in the equation $2^4 2^{3y} = \frac{1}{\sqrt{2}}$.

$2^4 2^{3y} = \frac{1}{\sqrt{2}}$

$2^{4+3y} = 2^{-1/2}$

so $4+3y = -\frac{1}{2}$ since $f(x)=2^x$ is one-to-one.

$4+3y = -\frac{1}{2}$

$3y = -9/2$

$y = -3/2$.

CHECK:

$2^4 \cdot 2^{3(-3/2)}$

$= 2^4 2^{-9/2}$

$= 2^{-1/2} = 1/\sqrt{2}$.

See p.276

2. (12 points) For any function of the form $f(x) = b^x$ with $0 < b < 1$, fill in the following blanks with intervals, values, or the word none.

(a) The domain of f is $(-\infty, \infty)$.

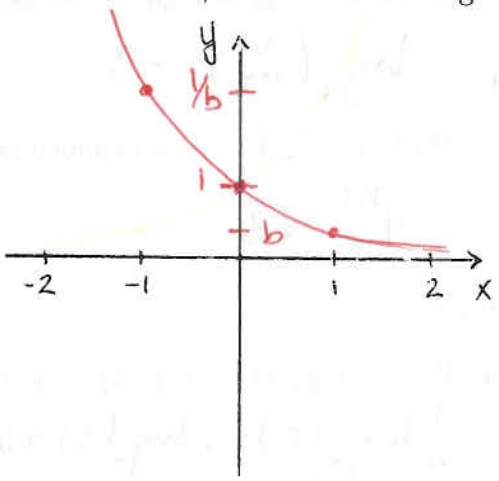
(b) The range of f is $(0, \infty)$.

(c) The x-intercept(s) of f is none.

(d) The y-intercept of f is 1.

(e) As $x \to \infty$, $f(x) \to$ 0.

(f) Sketch a graph of the function, making appropriate marks on the y-axis.

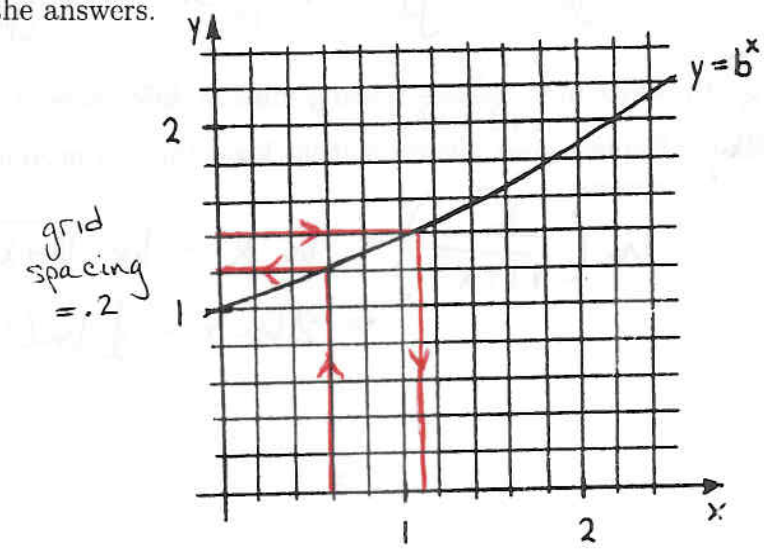


Compare to Sample Exam #5.

3. (4 points) The graph of an exponential function $y = b^x$ is sketched below. Use that graph to estimate $b^{0.6}$ and $\log_b(1.4)$, each rounded to one decimal place. Mark the graph below to show how you got the answers.

$b^{0.6} \approx$ 1.2

$\log_b(1.4) \approx$ 1.1



Same as Quiz 5 #1

4. (6 points) State the precise definition of a *logarithmic function*.

The logarithmic function $f(x) = \log_b(x)$ is the function whose output is the power to which we must raise b to yield x.

5. (8 points) True or false; no explanation necessary.

(a) $(70^{20})^5 = 70^{100}$ TRUE $(70^{20})^5 = 70^{20 \cdot 5} = 70^{100}$

(b) $\log_3 2 < 1$ TRUE To raise 3 to a power and get 2, the power is < 1.

(c) $\ln 2x^3 = 3 \ln 2x$ FALSE The power 3 applies only to x, not the 2.

(d) Doubling time for an account depends on the amount of money currently in the account. FALSE. Exponential growth means by a constant factor and P_0 drops out in solving.

6. (6 points)

Compare to Sample Exam #7.

(a) Write $4^{-3} = \frac{1}{64}$ in logarithmic form.

$\log_4\left(\frac{1}{64}\right) = -3$

(b) Write $\log_9(\sqrt{3}) = \frac{1}{4}$ in exponential form.

$9^{1/4} = \sqrt{3}$.

7. (10 points)

Compare to 5.4 #14,15

(a) Write $\frac{1}{3}\log_a(8) - 2\log_a(3) + \log_a(36)$ as a logarithm of a single number.

$\frac{1}{3}\log_a(8) - 2\log_a(3) + \log_a(36)$

$= \log_a 8^{1/3} - \log_a 3^2 + \log_a(36)$

$= \log_a 2 - \log_a 9 + \log_a 36 = \log_a \frac{2}{9} \cdot 36 = \log_a 8.$

This is 5.4 #216

(b) Write $\ln\left(\frac{x^2}{\sqrt{1+x^2}}\right)$ using sums or differences of simpler logarithmic expressions. Express your answer so that logarithms of products, quotients, or powers do not appear.

$\ln\left(\frac{x^2}{\sqrt{1+x^2}}\right) = \ln x^2 - \ln\sqrt{1+x^2}$

$= 2\ln x - \frac{1}{2}\ln(1+x^2)$

8. (10 points)

(a) Find the domain of $\log_6(1+5x)$.

$1+5x>0$

$5x>-1$

$x>-\frac{1}{5}$ so the domain is $(-\frac{1}{5},\infty)$.

Compare to web review #6.

(b) Solve the inequality $\log_6(1+5x) \le 2$.

$\log_6(1+5x) \le 2$

$1+5x \le 6^2 = 36$

$5x \le 36-1 = 35$

$x \le \frac{35}{5} = 7$ but we also need $x > -1/5$ so the solution set is $(-\frac{1}{5}, 7]$.

9. Solve the following equations and inequality. Give exact answers and show **all** steps by hand to receive full credit.

Compare to web review #5b.

(a) (8 points) $\log_2(x) + \log_2(3x+1) = 1$

$\log_2(x(3x+1)) = 1$

$\log_2(3x^2+x) = 1$

$3x^2+x = 2^1 = 2$

$3x^2+x-2=0$

$x = \frac{-1\pm\sqrt{1+24}}{6} = \frac{-1\pm 5}{6} = -1 \text{ or } \frac{2}{3}$.

But -1 is not in the domain of $\log_2(x)$ so only $x=2/3$ is a solution.

CHECK: $\log_2(2/3)+\log_2(2+1)$

$= \log_2\left(\frac{2}{3}\cdot 3\right) = \log_2 2 = 1$. ✓

This is 5.5 #1.

(b) (6 points) $5^x = 3^{2x-1}$

$\ln 5^x = \ln 3^{2x-1}$

$x\ln 5 = (2x-1)\ln 3 = (2\ln 3)x - \ln 3$

$x(\ln 5 - 2\ln 3) = -\ln 3$

$x = \frac{\ln 3}{2\ln 3 - \ln 5} = \frac{\ln 3}{\ln 3^2 - \ln 5} = \frac{\ln 3}{\ln(9/5)}$.

CHECK: $x \approx 1.869$

$5^{1.869} \approx 20.25$

$3^{2(1.869)-1} \approx 20.25$ ✓

Compare to 5.5 #48.

(c) (4 points) $4e^{-x} \ge 12$

$e^{-x} \ge \frac{12}{4} = 3$

$e^{-x} \ge 3$

$-x \ge \ln 3$

$x \le -\ln 3$.

10. (4 points) In what important context does the number e naturally arise?

The number e arises when computing compound interest and the number of compounding periods becomes arbitrarily large.
($\left(1+\frac{1}{n}\right)^n \to e$ as $n \to \infty$ and $\left(1+\frac{r}{n}\right)^n \to e^r$ as $n \to \infty$.)

11. (6 points) A sum of \$1000 is placed in a savings account at a nominal interest rate of 7% per annum. How much is in the account after 20 years if interest is compounded quarterly and no additional deposits or withdrawals are made?

Use $P(t) = P_0\left(1+\frac{r}{n}\right)^{nt}$.

$P(20) = 1000\left(1+\frac{.07}{4}\right)^{4\cdot 20}$

$= 1000\,(1.0175)^{80}$

$\approx \$4006.39$ is in the account after 20 years.

This is 5.6 #7b.

12. (10 points) The half-life of plutonium-241 is 13 years. How much of an initial 2 g sample remains after 5 years? Give your answer as a decimal approximation rounded to two decimal places.

$\frac{1}{2} = e^{k\cdot 13}$ so $\ln\left(\frac{1}{2}\right) = 13k$ and $k = \frac{1}{13}\ln(1/2)$.

So $N(t) = N_0 e^{\frac{1}{13}\ln(1/2)t}$

and $N(5) = 2e^{\frac{1}{13}\ln(\frac{1}{2})\cdot 5}$

≈ 1.53 grams remain after 5 years.

This is 5.7 #27a.