College Algebra, Math 143, Kriloff
Exam 1, Fall 2002

Instructions: Show your work! If you use a calculator, write what you computed or sketch the graph you used. Include intermediate steps. Multiple answers or a correct answer without a reasonable amount of work might receive no credit. Give exact answers unless asked for decimal approximations. When finished, check your work by hand or on your calculator.

1. (10 points) Let $f(x) = 3x^2 - 2$.

3 (a) Find $f(1 - \sqrt{2})$. Give an *exact* value and simplify your answer.

$f(1-\sqrt{2}) = 3(1-\sqrt{2})^2 - 2$
$= 3(1-2\sqrt{2}+2)-2 = 3-6\sqrt{2}+6-2 = 7-6\sqrt{2}.$

3 (b) Find $f(-2x) + 1$. Simplify your answer.

$f(-2x)+1 = 3(-2x)^2-2+1 = 3\cdot 4x^2-2+1$
$= 12x^2-$

4 (c) Find the average rate of change of f on the interval $[-1, 2]$.

$\frac{\Delta f}{\Delta x} = \frac{f(2)-f(-1)}{2-(-1)} = \frac{3\cdot 2^2-2-(3(-1)^2-2)}{3} = \frac{12-2-(3-2)}{3}$
$= \frac{9}{3} = 3.$

2. (10 points) Find the domain of each function. Give your answer in interval notation.

2 (a) $f(x) = -x^2 - 3x + 10$

The domain of f (and all polynomials) is $(-\infty, \infty)$.

3 (b) $f(x) = \frac{1}{-x^2 - 3x + 10} = \frac{1}{(2-x)(5+x)}$ or $\frac{-1}{(x-2)(x+5)}$

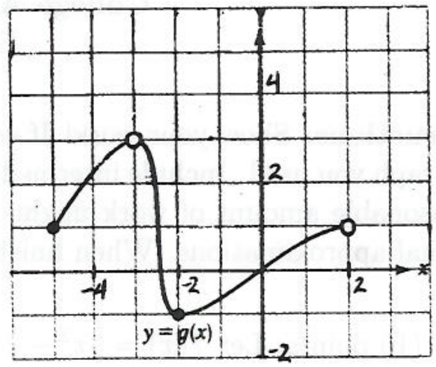
The domain of f is all values of x so the denominator is not 0 : $x \neq 2$ or -5 or $(-\infty,-5)\cup(-5,2)\cup(2,\infty)$.

5 (c) $f(x) = \sqrt{-x^2 - 3x + 10}$ Solve $-x^2-3x+10 = (2-x)(x+5)$ using a sign chart:

The domain is $[-5,2]$

	$(-\infty,-5)$	-5	$(-5,2)$	2	$(2,\infty)$
$2-x$	+		+		−
$x+5$	−		+		+
$-x^2-3x+10$	−		+		−

3. (9 points) The graph of $g(x)$ is shown.



2 (a) Give the minimum value of g.

-1

2 (b) Give the range of g.

$[-1,3)$

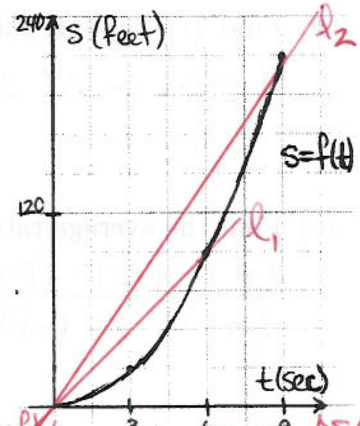
4 (c) For which value(s) of x is $g(x) = 1$?

-5 and ≈ -2.5

1 (d) Give the interval(s) on which g is increasing.

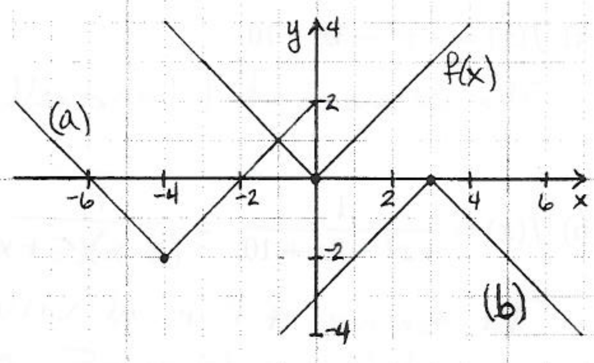
$(-5,-3)$ and $(-2,2)$

4. (5 points) The graph of the distance s travelled by a falling ball is shown below. Is the average velocity of the ball, $\frac{\Delta s}{\Delta t}$ greater on the interval $[0, 6]$ or on the interval $[0, 9]$? Explain and/or show how you can tell.



The average velocity on an interval is the slope of the line through the two points on the distance function at the endpoints of the interval. Since on the graph line l_2 is steeper than line l_1, we know the average velocity is greater on the interval $[0,9]$ than on the interval $[0,6]$. Or find on $[0,6]$, $\Delta s/\Delta t = 96/6 = 16$ ft/sec and on $[0,9]$, $\Delta s/\Delta t = 216/9 = 24 \frac{ft}{sec}$.

5. (8 points) Give formulas for the indicated graphs. Use both general function notation and give a specific formula.



3+1 (a) $f(x+4)-2 = |x+4|-2$

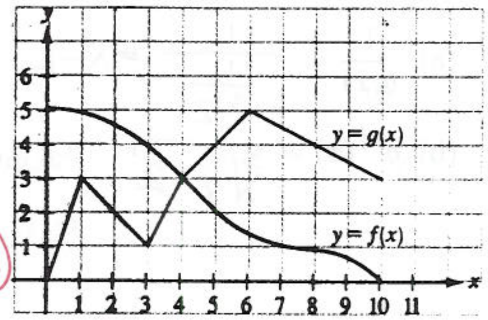
3+1 (b) $-f(x-3) = -|x-3|$

6. (5 points) Explain carefully in words how to obtain the graph of $f(-x + 5)$ from the graph of $f(x)$.

Reflect the graph of $f(x)$ in the y-axis and then shift the result right 5 units, or shift the graph of $f(x)$ left 5 units and reflect the result in the y-axis. (Try graphing $(-x+5)^3$ for instance and compare to $y=x^3$.)

7. (12 points) Find the following using the graphs of f and g given below.

3 each (a) $(g+f)(4)$ $= g(4)+f(4) = 3+3 = 6$



(b) Value(s) of x with $(f-g)(x) \leq 0$.

$[4,10]$ (Look where f is at or below g on the graph.)

(c) $f(g(6))$ $= f(5) = 2$

(d) $(g \circ g)(8)$ $= g(g(8)) = g(4) = 3$

8. (3 points) Find $(H \circ K)(0)$ or state that it is undefined.

x	-2	0	3	5	6
$H(x)$	1	4	2.5	0	-3

x	-4	-2	0	1	4
$K(x)$	1/3	3	-2	0	6

$(H \circ K)(0) = H(K(0)) = H(-2) = 1$

9. (8 points) Let $k(x) = 10$, $g(x) = x+1$, and $h(x) = \dfrac{3-x}{x^2}$.

4 (a) Find $h(g(x))$ **and** give its domain.

2 $h(g(x)) = h(x+1) = \dfrac{3-(x+1)}{(x+1)^2} = \dfrac{2-x}{(x+1)^2}$ has domain $x \neq -1$ or $(-\infty,-1)\cup(-1,\infty)$.

2 (b) Find $k(h(x))$.

$k(h(x)) = k\left(\dfrac{3-x}{x^2}\right) = 10$ (since k is the constant function 10, its output is always 10, regardless of the input).

(Compare to 3.4 #14d.)

10. (4 points) Find $f(x)$ and $g(x)$ so that $f(g(x)) = -5\sqrt[3]{x}+7$. (Your $f(x)$ and $g(x)$ cannot be just x.)

$g(x) = \sqrt[3]{x}$, $f(x) = -5x+7$ or $g(x) = -5\sqrt[3]{x}$, $f(x) = x+7$ or $g(x) = 5\sqrt[3]{x}$, $f(x) = -x+7$

11. (8 points) Let $g(x) = \dfrac{1}{x^3 - 2}$.

2 (a) $\dfrac{1}{g(x)} = \dfrac{1}{\frac{1}{x^3-2}} = x^3 - 2$

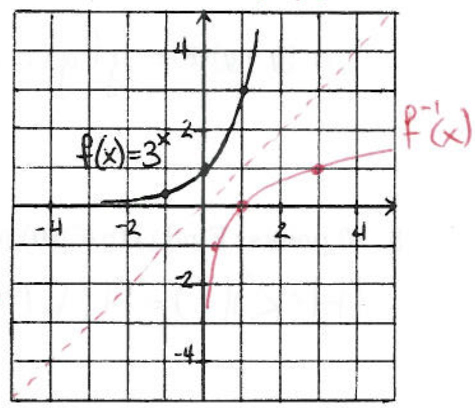
6 (b) $g^{-1}(x) = \sqrt[3]{\frac{1}{x}+2}$ Swap x and y: $x = \dfrac{1}{y^3-2}$

Solve for y: $y^3 - 2 = \frac{1}{x}$

$y^3 = \frac{1}{x} + 2$

$y = \sqrt[3]{\frac{1}{x}+2}$ or $= \sqrt[3]{\frac{1+2x}{x}}$.

12. (9 points) The graph of $f(x) = 3^x$ is shown.



3 (a) Explain how you can tell from the graph that f has an inverse function. (The x-axis is an asymptote.)

Since no horizontal line hits the graph of f(x) more than once, f is one-to-one and has an inverse.

4 (b) Carefully sketch the graph of f^{-1} on the same axes.

2 (c) Give the domain of f^{-1}. The domain of f^{-1} is the range of f, $(0, \infty)$

13. (a) (3 points) Complete the sentence to make a correct and precise statement.
A function f is one-to-one if and only if

for all a and b in the domain of f, if f(a) = f(b), then a = b.
OR no horizontal line hits the graph of f more than once.
OR f has an inverse function.

(b) (2 points) Give an example of a function that is not one-to-one. $f(x) = x^2$
(There are several correct answers.) $f(x) = |x|$ etc.

14. (4 points) Let $G(t)$ be the temperature in Celsius of a solution t minutes into a chemistry experiment. Assuming that $G(t)$ is a one-to-one function for $0 \le t \le 10$, use the table to estimate $G^{-1}(25)$ and state what this number means in practical terms.

t (mins)	0	3	6	8	10
$G(t)$ (°C)	22	23	27	28	28.5

$G^{-1}(25)$ would appear to be somewhere between 3 and 6, so maybe around 4.5 minutes. This is the time the solution reaches 25°C.
(Compare to Sample Exam 1 #14.)