College Algebra, Math 143, Kriloff
Exam 2, Spring 2006

Instructions: Show your work! If you use a calculator, write what you computed or sketch the graph you used. Include intermediate steps. Always give units for answers to applied problems. Multiple answers or a correct answer without a reasonable amount of work might receive no credit. Give exact answers unless asked for decimal approximations. When finished, check your work by hand or on your calculator.

1. (8 points) Find the linear function such that $f(2) = -5$ and the graph of f is a line parallel to $x + y = 4$.

$x+y=4$ is the same as $y=-x+4$ so we want slope of -1
We are given the point $(2,-5)$.

$y-(-5)=-1(x-2)$
$y+5=-x+2$
$y=-x-3$

2. (12 points) The data in the table shows the U.S. national debt, $f(t)$, measured in billions of dollars t years after 1985. The regression line that models the data is $f(t) = 258.5t + 2018$.

(a) Use the regression line to predict the U.S. national debt in 2005.

$t = 2005 - 1985 = 20$
$f(20) = 258.5 \cdot 20 + 2018$
$= 7188$ billion dollars

t in years after 1985	$f(t)$ in billions of \$
0	1945.9
5	3233.3
10	4974.0
15	5674.2

(b) Use that the actual debt in 2005 was 7932.7 billion dollars to find the percentage error in your prediction. Be sure to check if your answers are reasonable!

$$\frac{|\text{actual} - \text{estimate}|}{\text{actual}} \cdot 100 = \frac{|7932.7 - 7188|}{7932.7} = \frac{744.7}{7932.7} \approx 9.4\%$$

3. (6 points) Complete the square for $y = 4x^2 + 48x + 31$.

$y = 4(x^2+12x+\quad) \quad +31$

$y = 4(x^2+12x+36) - 4\cdot 36 + 31 = 4(x^2+12x+6^2) - 144 + 31$

$y = 4(x+6)^2 - 113$

4. (12 points) If $f(x) = -2(x+\pi)^2 + \frac{2}{7}$, find the following or state there is none.

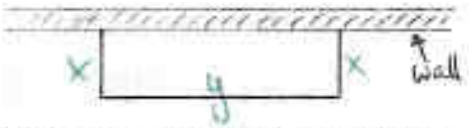
(a) The exact maximum or minimum value of the function (circle which one).

$2/7$

(b) Describe carefully how to obtain the graph of $f(x)$ from the graph of x^2.

Shift $y=x^2$ left π units,
then stretch vertically by a factor of 2,
then reflect in the x-axis,
then shift up $2/7$.

5. We want to enclose a rectangular region alongside a wall using 60 feet of fence as shown in the figure.



(a) (6 points) Express the area of the region as a function of the length of one of the sides perpendicular to the wall.

Let x be the length of the sides perpendicular to the wall and y be the length of the other side. Key quantity: $A = xy$.
Since $2x+y=60$, $y = 60-2x$ so
$A = xy = x(60-2x)$

(b) (6 points) Find the dimensions that maximize the area.

$A = 60x - 2x^2$ or $A = -2x^2 + 60x$
The vertex with maximum A value occurs when
$x = -\frac{b}{2a} = \frac{-60}{2(-2)} = \frac{-60}{-4} = 15$ feet
Then the other dimension is $y = 60 - 2\cdot 15 = 60 - 30 = 30$ feet.

6. (7 points) Express the circumference of a circle as a function of its area.

Circumference is $C = 2\pi r$ (linear or one-dimensional measurement)
Area of a circle is $A = \pi r^2$ (two-dimensional measurement)
Solve for r in terms of A and substitute in C:
$r^2 = A/\pi$ so $r = \sqrt{A/\pi}$ (since $r > 0$ we use only the positive root)
$C = 2\pi \cdot \sqrt{A/\pi}$ or $C = 2\sqrt{\pi}\sqrt{A}$

7. (19 points) For $f(x) = (x+3)^2(x-\sqrt{2})(1-x)$ find the following.

(a) Exact x-intercepts with a decimal approximation as well if appropriate.

$-3, 1, \sqrt{2} \approx 1.4$ (values that make a factor zero)

(b) Exact y-intercept with a decimal approximation as well if appropriate.

$f(0) = (0+3)^2(0-\sqrt{2})(1-0) = 3^2(-\sqrt{2})(1) = -9\sqrt{2} \approx -12.7$

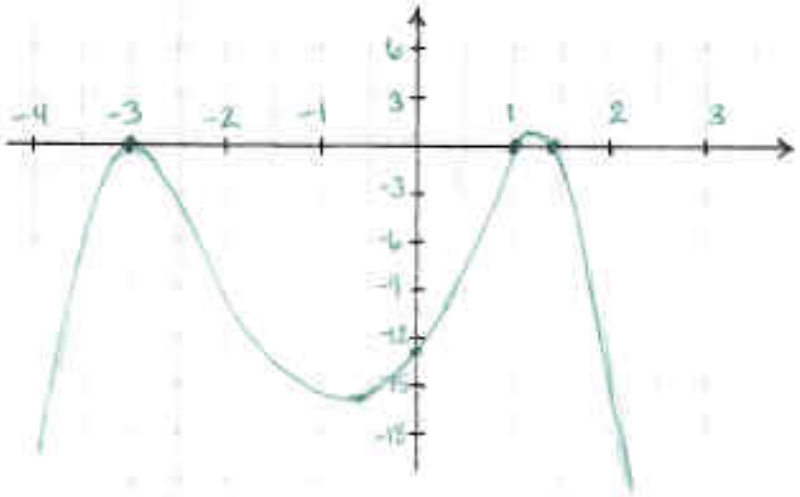
(c) The degree of $f(x) = 4$

The leading coefficient of $f(x) = -1$ } leading term if multiplied out would be $-x^4$

(d) Complete a sign chart.

	-3		1	$\sqrt{2} \approx 1.4$
$x - \sqrt{2}$	$-$	$-$	$-$	$+$
$1 - x$	$+$	$+$	$-$	$-$
$f(x)$	$-$	$-$	$+$	$-$

(e) Sketch a graph of $f(x)$. Choose and mark scales that show relevant behavior.



8. For $f(x) = \dfrac{2x^2+3x-5}{4x-7x^2} = \dfrac{(2x+5)(x-1)}{x(4-7x)}$ find the following by hand or state there is none. Show work!

(a) (4 points) The domain of f.

$x \neq 0, \frac{4}{7}$ so $(-\infty, 0) \cup (0, \frac{4}{7}) \cup (\frac{4}{7}, \infty)$

(b) (4 points) Exact x-intercepts, if any.

$x-1=0$ at $x=1$

$2x+5=0$ if $2x=-5$ or $x=-5/2$ so 1 and $-5/2=-2.5$

(c) (2 points) Exact y-intercept, if any.

None (setting $x=0$ makes the denominator 0 and y undefined)

(d) (4 points) Exact equations of any vertical asymptotes.

$x=0$

$x=\frac{4}{7}$ (since $4-7x=0$ if $7x=4$ or $x=4/7$)

(e) (4 points) Exact equation of any horizontal asymptote.

$$\frac{(2x^2+3x-5)1/x^2}{(4x-7x^2)1/x^2} = \frac{2+3/x-5/x^2}{4/x-7} = \frac{2+3/x-5/x^2}{-7+4/x} \approx \frac{-2}{7} \text{ if } |x| \text{ is large}$$

so $y=-\frac{2}{7}$

(f) (6 points) Sketch a graph of $f(x)$. Choose and mark scales that show relevant behavior.