College Algebra, Math 143, Kriloff
Exam 1, Spring 2006

Instructions: Show your work! If you use a calculator, write what you computed or sketch the graph you used. Include intermediate steps. Always give units for answers to applied problems. Multiple answers or a correct answer without a reasonable amount of work might receive no credit. Give exact answers unless asked for decimal approximations. When finished, check your work by hand or on your calculator.

1. (15 points) Find the domain of each function in interval notation. Show your work!

(a) $f(x) = \dfrac{1}{2x^2 - 5x - 3} = \dfrac{1}{(2x+1)(x-3)}$

The denominator is 0 when $x-3=0$ or $x=3$ and when $2x+1=0$ or $x=-\frac{1}{2}$

So the domain is all real numbers except $x=-\frac{1}{2}$ and $x=3$, or $(-\infty,-\frac{1}{2})\cup(-\frac{1}{2},3)\cup(3,\infty)$.

(b) $f(x) = \sqrt{2x^2 - 5x - 3} = \sqrt{(2x+1)(x-3)}$

Test values on these intervals to see where $2x^2-5x-3 \geq 0$:

pos. | neg. | pos.

$-1/2$ 3

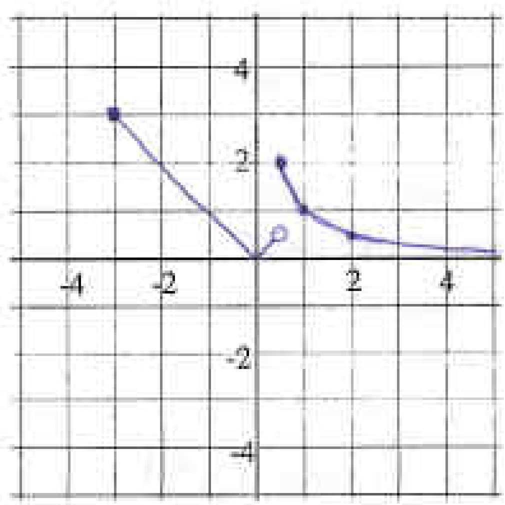
so the domain is $(-\infty,-\frac{1}{2}]\cup[3,\infty)$

(c) $f(x) = \sqrt[3]{2x^2 - 5x - 3}$

Domain: $(-\infty,\infty)$

2. (10 points)

(a) Sketch the graph of
$$g(x) = \begin{cases} |x|, & \text{if } -3 \leq x < \frac{1}{2} \\ 1/x, & \text{if } \frac{1}{2} \leq x. \end{cases}$$



(b) Does g have an inverse? Explain why or why not.

No, g does not have an inverse because its graph fails the horizontal line test. (For example, the line $y=2$ intersects the graph twice.)

3. Let $f(x) = x^3 - 5$.

(a) (4 points) Show $f(-1) + f(2) \neq f(-1+2)$.

$f(-1) + f(2) = (-1)^3 - 5 + 2^3 - 5 = -1 - 5 + 8 - 5 = -6 + 3 = -3$

$f(-1+2) = f(1) = 1^3 - 5 = 1 - 5 = -4$ and $-3 \neq -4$.

(b) (6 points) Find each of the following and simplify your answer if possible.

$\frac{1}{f(x)} = \frac{1}{x^3-5}$ $\qquad$ $f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^3 - 5 = \frac{1}{x^3} - 5$ or $x^{-3} - 5$

$f(2x) = (2x)^3 - 5 = 8x^3 - 5$ $\qquad$ $f(x) + 3 = x^3 - 5 + 3 = x^3 - 2$

(c) (3 points) Find x so that $f(x) = 22$.

$x^3 - 5 = 22$

$x^3 = 22 + 5 = 27$ so $x = \sqrt[3]{27} = 3$. CHECK: $3^3 - 5 = 27 - 5 = 22$ ✓

(d) (6 points) Find $f^{-1}(x)$ and use algebra to check that your answer is correct.

$y = x^3 - 5$

$x = y^3 - 5$

$y^3 = x + 5$

$y = \sqrt[3]{x+5}$ so $f^{-1}(x) = \sqrt[3]{x+5}$

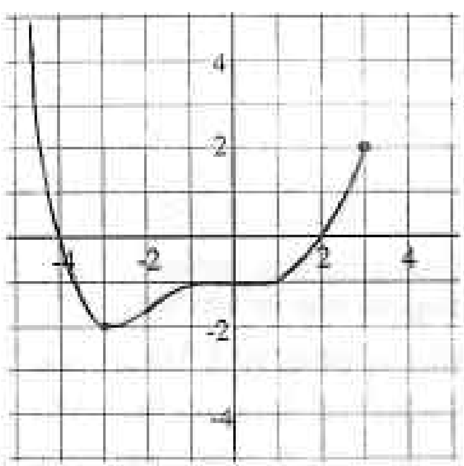
CHECK: $f(f^{-1}(x)) = f(\sqrt[3]{x+5})$

$= (\sqrt[3]{x+5})^3 - 5 = (x+5) - 5 = x$ ✓

$f^{-1}(f(x)) = f^{-1}(x^3 - 5)$

$= \sqrt[3]{(x^3-5)+5} = \sqrt[3]{x^3} = x$ ✓

4. (8 points) The graph of $g(x)$ is shown. Give the following or state that they do not exist.



(a) The minimum value of g.

-2

(b) The maximum value of g.

Does not exist

(c) The range of g.

$[-2, \infty)$

(d) The interval(s) on which g is increasing.

$(-3, -1)$ and $(1, 3)$

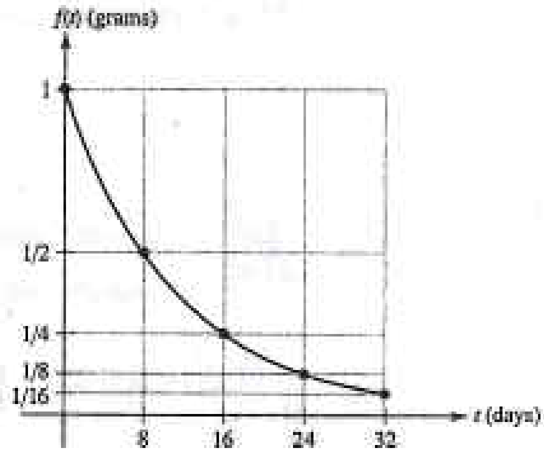
5. (8 points) The graph shows how an initial one-gram sample of the radioactive substance Iodine-131 decays over a 32-day period. The value $f(t)$ represents the number of grams present after t days.
Find the average rate of change in the number of grams of Iodine-131 on the interval from $t = 8$ days to $t = 24$ days. Start by using the function notation $f(t)$ to write the difference quotient $\frac{\Delta f}{\Delta t}$. Give an exact value and a decimal approximation.

$$\frac{\Delta f}{\Delta t} = \frac{f(24)-f(8)}{24-8} = \frac{\frac{1}{8}-\frac{1}{2}}{16}$$

$$= \frac{1}{16}\left(\frac{1}{8}-\frac{4}{8}\right)$$

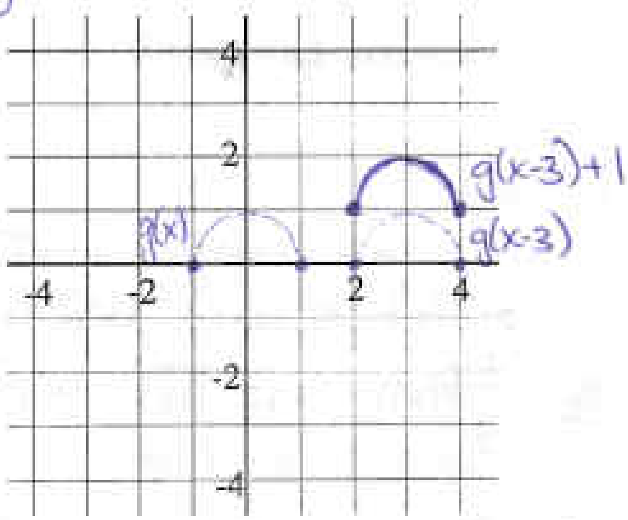
$$= \frac{1}{16}\left(-\frac{3}{8}\right)$$

$$= \frac{-3}{128} \approx -.023 \ \frac{\text{grams}}{\text{day}}$$



6. (6 points) If $g(x) = \sqrt{1-x^2}$, draw the graph of $g(x-3)+1$.

For the graph of $g(x-3)+1$, shift the graph of $g(x)$ right 3 units and then up 1.



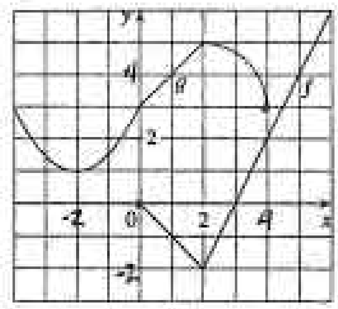
7. (9 points) For an arbitrary function $F(x)$, explain carefully in words how to obtain the graph of each given function from the graph of $F(x)$.

(a) $F(x+4)$ Shift the graph of F(x) left 4 units to get the graph of F(x+4).

(b) $-F(x)-2$ Reflect the graph of F(x) in the x-axis, then shift down by 2 units,

OR Since $-F(x)-2 = -(F(x)+2)$, shift the graph of F(x) up 2 units, then reflect in the x-axis, to get the graph of $-F(x)-2$.

8. (14 points) Using the graphs of f and g, find the following or state that they do not exist and explain why not.



(a) $(fg)(1)$ $= f(1)g(1) = -1 \cdot 4 = -4$

(b) $(f-g)(-2)$ does not exist since $f(-2)$ is not defined

(c) The domain of $f-g$. $[0,4]$ or $0 \le x \le 4$

(d) $(g/f)(3)$ $= \frac{g(3)}{f(3)}$ does not exist since $g(3) \approx 4.7$, but $f(3)=0$ and we cannot divide by 0.

(e) $(g \circ f)(2)$ $= g(f(2)) = g(-2) = 1$

(f) Values of x where $f(x) \le 0$. $[0,3]$ or $0 \le x \le 3$

9. (7 points)

(a) If $f(x) = 3x+2$ and $g(x) = 1+\sqrt{2-x}$, find $g(f(x))$ and simplify.

$g(f(x)) = g(3x+2) = 1+\sqrt{2-(3x+2)} = 1+\sqrt{2-3x-2} = 1+\sqrt{-3x}$.

OR $g(f(x)) = 1+\sqrt{2-f(x)} = 1+\sqrt{2-(3x+2)} = 1+\sqrt{2-3x-2} = 1+\sqrt{-3x}$.

(b) Find $f(x)$ and $g(x)$ so that $f(g(x)) = \frac{1}{1+2x^4}$. (Do not use just x.)

$f(x) = \frac{1}{x}$ and $g(x) = 1+2x^4$ CHECK: $f(g(x)) = f(1+2x^4) = \frac{1}{1+2x^4}$ ✓

OR $f(x) = \frac{1}{1+x}$ and $g(x) = 2x^4$ CHECK: $f(g(x)) = f(2x^4) = \frac{1}{1+2x^4}$ ✓

10. (4 points) Let $G(t)$ be the temperature in Celsius, C, of a solution t minutes into a chemistry experiment. Assuming that $G(t)$ is a one-to-one function for $0 \le t \le 10$, use the table to find $G^{-1}(6)$ and state what this number means in practical terms.

t (mins)	0	3	6	9	12
$G(t)$ (C)	15	12	10	9	6

$G^{-1}(6) = 12$ minutes. This is the time when the solution reaches a temperature of 6C.