



## MARDEN'S THEOREM

**ABSTRACT.** Let  $p$  be a polynomial with complex coefficients. The Gauss-Lucas Theorem states that the roots of  $p'$ , the complex derivative of  $p$ , lie in the convex hull of the roots of  $p$ . The simplest version of Morris Marden's Theorem is a sharp refinement of the Gauss-Lucas Theorem for cubic polynomials. It states that if the roots  $z_i$  of  $p$  are noncollinear and  $T$  is the triangle having the  $z_i$  as its vertices, then the roots of  $p'$  are the foci  $w_i$  of the unique ellipse that is tangent at the midpoints of the sides of  $T$ .

The proof of Marden's Theorem involves elementary aspects of group actions, the mirror principle from optics that characterizes when a line is tangent to an ellipse and also the argument function from complex function theory. The talk will detail the main aspects of the proof of the theorem and will illustrate it using Geometer's Sketchpad constructions.