

Indexing regions in dihedral and dodecahedral hyperplane arrangements

Cathy Kriloff

Dominant regions in hyperplane arrangements and associated combinatorial objects have appeared in many contexts within the last twenty years; for instance in the contexts of Kazhdan-Lusztig cells for affine Weyl groups (Shi), combinatorics of non-nesting partitions and ad-nilpotent ideals in Lie algebras (Reiner, Postnikov, Cellini-Papi, Sommers, Athanasiadis, Panyushev) and unitary representations of real Lie groups and p -adic algebraic groups and their Hecke algebras (Barbasch, Vogan, Ciubotaru).

This work has shown that for certain crystallographic arrangements of reflecting and affine hyperplanes, regions in the hyperplane arrangement are in bijection with simpler combinatorial objects. In this talk examples will illustrate how to index dominant regions by ideals, or equivalently their generating sets, called antichains, in a partial order on vectors orthogonal to the reflecting hyperplanes.

The main goal is to discuss our result that for the noncrystallographic dihedral (type $I_2(m)$) and dodecahedral (type H_3) cases, a similar bijection can be used to index the dominant regions. Surprisingly, the bijection breaks down in the four-dimensional hyperdodecahedral case (type H_4). We give a criterion that explains this failure, and provide a description of the image of the map from regions to antichains in the H_4 case. This work is joint with Yu Chen.