Metric foliations on hyperbolic spaces

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On the hyperbolic space D^n, codimension-one totally geodesic foliations of class C^k are classified. Except for the unique parabolic homogeneous foliation, the set of all such foliations is in one-one correspondence (up to isometry) with the set of all functions $z: [0,\pi] \to \partial D^n$ of class C^{k-1} with $z(0) = e_1 = z(\pi)$ satisfying

$$|z'(r)| \leq 1$$

for all r, modulo an isometric action by $\mathrm{O}(n-1) \times \mathbb{R} \times \mathbb{Z}_2$.

Since 1-dimensional metric foliations on D^n are always either homogeneous or flat (that is, their orthogonal distributions are integrable), this classifies all 1-dimensional metric foliations as well.

Equations of leaves for a non-trivial family of metric foliations on D^2 (called "fifth-line") are found.