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Colloquium Title: *Lie Sphere Geometry and Dupin Hypersurfaces*

Abstract: In his doctoral dissertation, published in *Mathematischen Annalen* in 1872, Sophus Lie introduced his geometry of oriented hyperspheres in Euclidean space $\mathbf{R}^n$ in the context of his work on contact transformations. Lie established a bijective correspondence between the set of all oriented hyperspheres, oriented hyperplanes and point spheres in $\mathbf{R}^n \cup \{\infty\}$, and the set of all points on the quadric hypersurface Q^{n+1} in real projective space $\mathbf{P}^{n+2}$ given by the equation $\langle x, x \rangle = 0$, where $\langle , \rangle$ is an indefinite scalar product with signature $(n+1, 2)$ on $\mathbf{R}^{n+3}$.

In this talk, we give Lie's construction in detail, and discuss its applications to the modern study of Dupin hypersurfaces, which are hypersurfaces in $\mathbf{R}^n$ with the property that each principal curvature function is constant along each leaf of its corresponding principal foliation. Examples of Dupin hypersurfaces are the images under stereographic projection of isoparametric (constant principal curvatures) hypersurfaces in the sphere S^n, including the cyclides of Dupin in $\mathbf{R}^3$.